Question: What will happen on Wed 2/21 at 8:00 P.M in ELLIOTT?

Exam I: **Lessons 1 to 17**.

Lesson 18 **Section 14.8**

(not on the exam).

Find the maximum and minimum values of

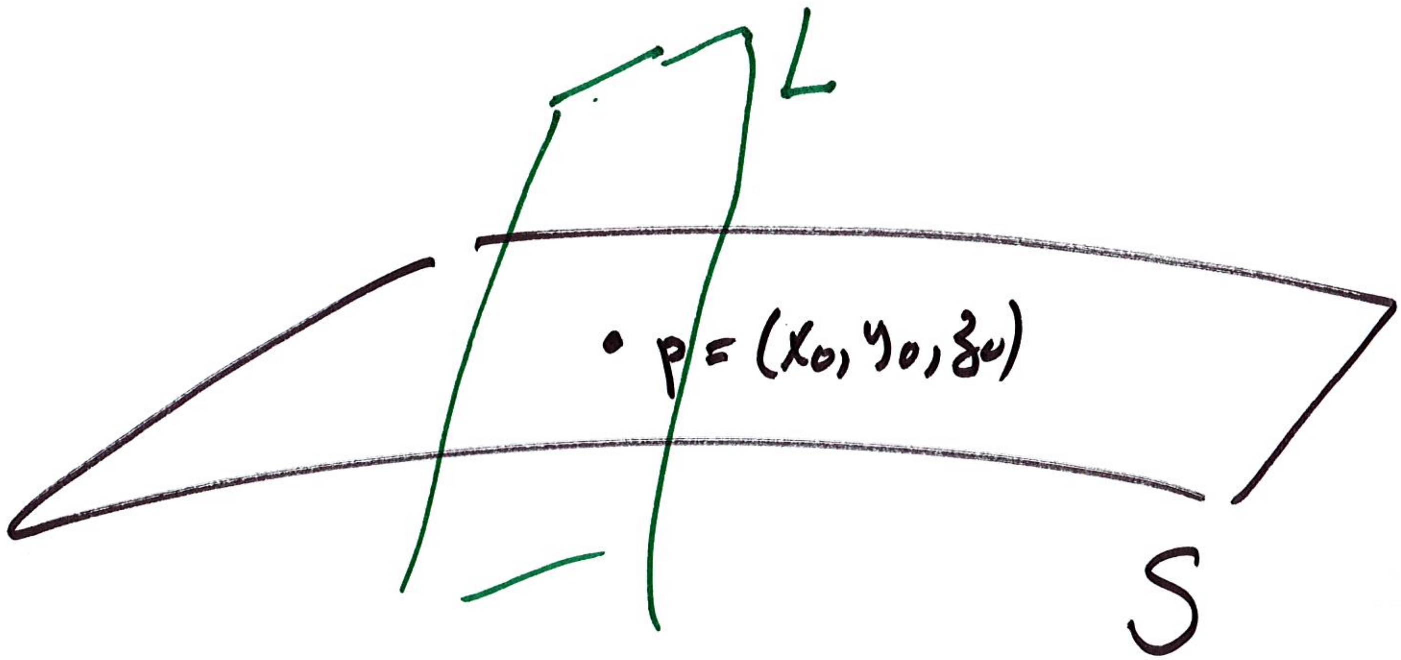
$$f(x,y,z)$$

on a surface

$$S = \{ \varphi(x,y,z) = c \}$$

$$\{\varphi = c\} = S$$

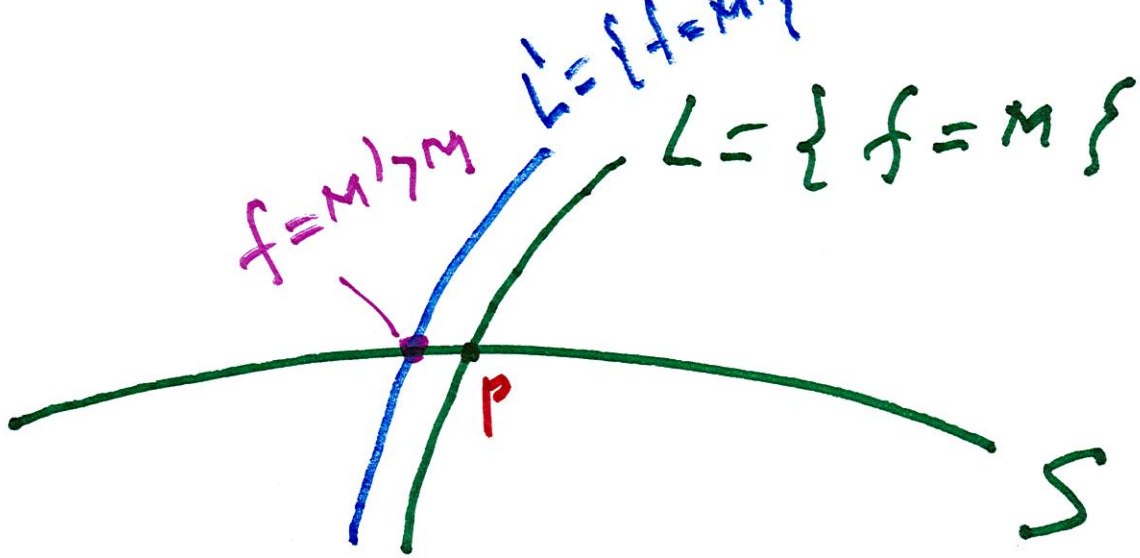
Lagrange Multipliers



Suppose $p = (x_0, y_0, z_0)$ is the maximum of f on S!

$$f(x,y,z) = M = f(x_0, y_0, z_0)$$

$$\{ f(x,y,z) = M \} = L$$



$M' > M$

If f is differentiable and of course continuous

$$L' = \{ f = M' \}$$

λ = Lagrange Multiplier.

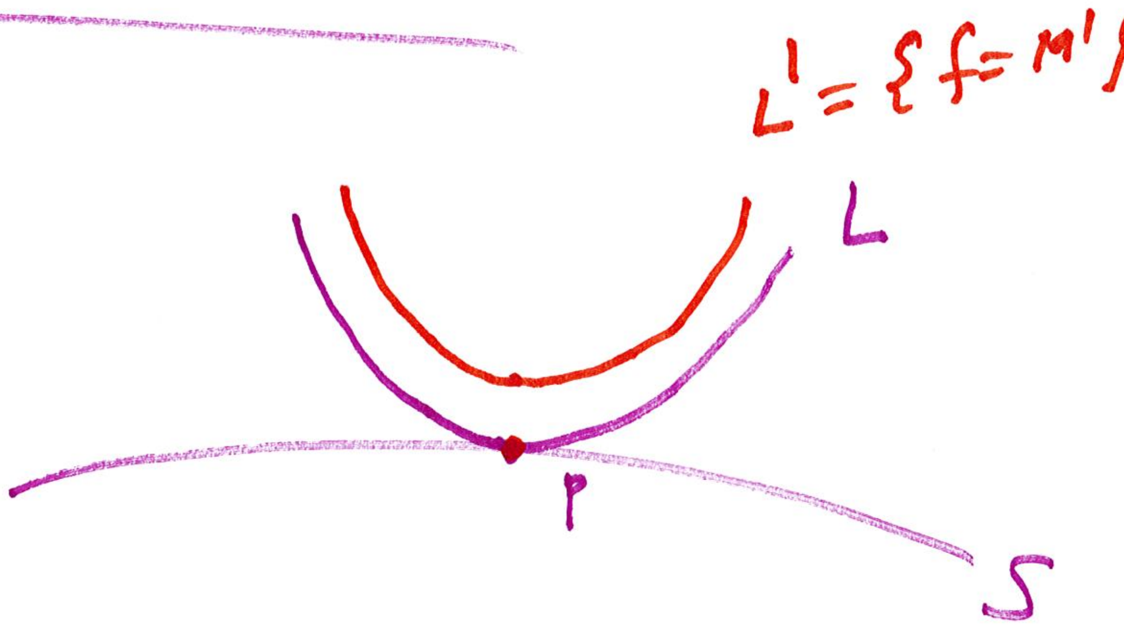
Strategy: If we want to find maximum and minimum of $f(x,y,z)$ on a surface

$$S = \{ g(x,y,z) = c \}$$

We find the points (x,y,z) that satisfy

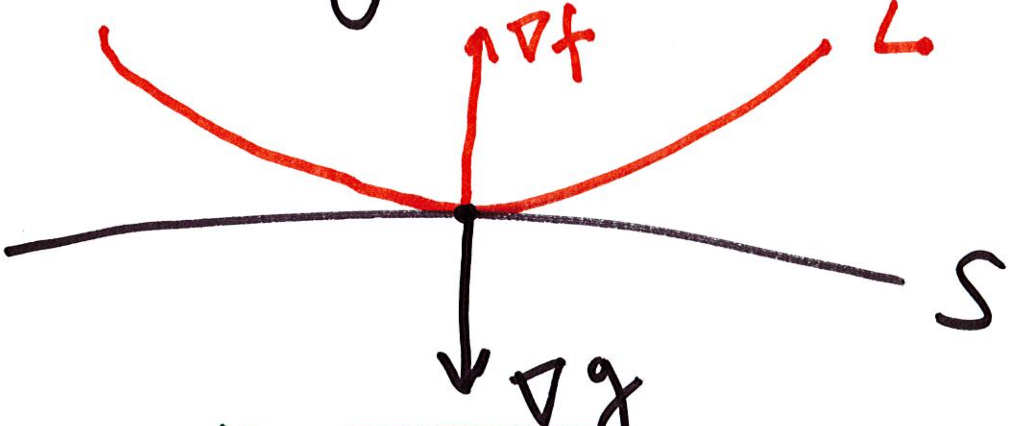
$$\begin{cases} \nabla f(x,y,z) = \lambda \nabla g(x,y,z) \\ g(x,y,z) = c \end{cases}$$

Conclusion:



If $S = \{ g(x,y,z) = c \}$

and $\nabla g(x,y,z) \neq 0$



Conclusion: $\boxed{\nabla f(p) = \lambda \nabla g(p)}$

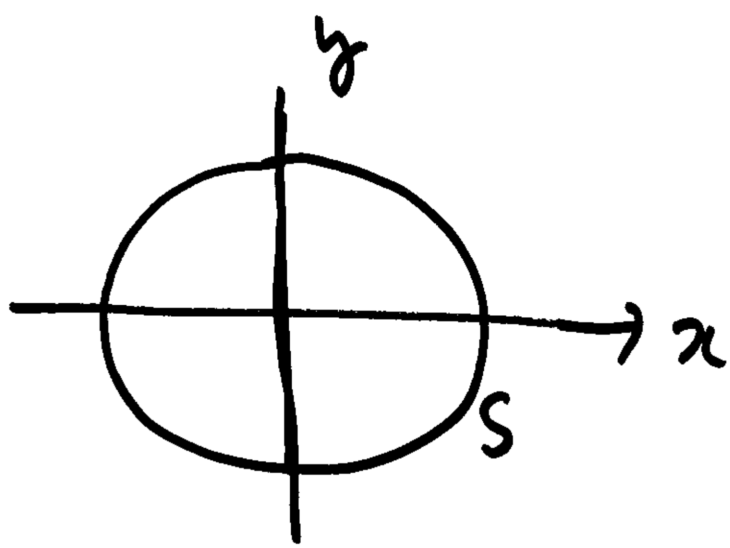
Example I: Find the maximum and minimum

of

$$f(x,y) = xy$$

on

$$\{x^2+y^2=3\} = S$$



Compute $\nabla f = \langle y, x \rangle$

$$g = x^2+y^2 \qquad \nabla g = \langle 2x, 2y \rangle$$

$$\nabla f = \lambda \nabla g$$

$$\nabla f = \langle y, x \rangle \; ; \quad \nabla g = \langle 2x, 2y \rangle$$

$$\begin{cases} \langle y, x \rangle = \lambda \langle 2x, 2y \rangle \\ x^2 + y^2 = 3 \end{cases}$$

$$\begin{cases} y = 2\lambda x \\ x = 2\lambda y \\ x^2 + y^2 = 3 \end{cases}$$

If $x \neq 0$, $y \neq 0$

$$y = 2\lambda x$$

$$x = 2\lambda y$$

$$\frac{y}{x} = \frac{2\lambda x}{2\lambda y} = \frac{x}{y}$$

$$\frac{y}{x} \qquad \frac{x}{y}$$

$$\boxed{y^2 = x^2}$$

But $x^2 + y^2 = 3$. $\qquad 2x^2 = 3$

$$x^2 = 3/2 \qquad \boxed{x = \pm\sqrt{3/2}}$$

$$y = \pm\sqrt{3/2}.$$

$\therefore \left(\sqrt{\tfrac{3}{2}}, \sqrt{\tfrac{3}{2}}\right); \left(-\sqrt{\tfrac{3}{2}}, \sqrt{\tfrac{3}{2}}\right)$

$\left(-\sqrt{\tfrac{3}{2}}, -\sqrt{\tfrac{3}{2}}\right); \left(\sqrt{\tfrac{3}{2}}, -\sqrt{\tfrac{3}{2}}\right)$

$f(x,y) = xy$

$f\left(\sqrt{\tfrac{3}{2}}, \sqrt{\tfrac{3}{2}}\right) = f\left(-\sqrt{\tfrac{3}{2}}, -\sqrt{\tfrac{3}{2}}\right) = \tfrac{3}{2}$.

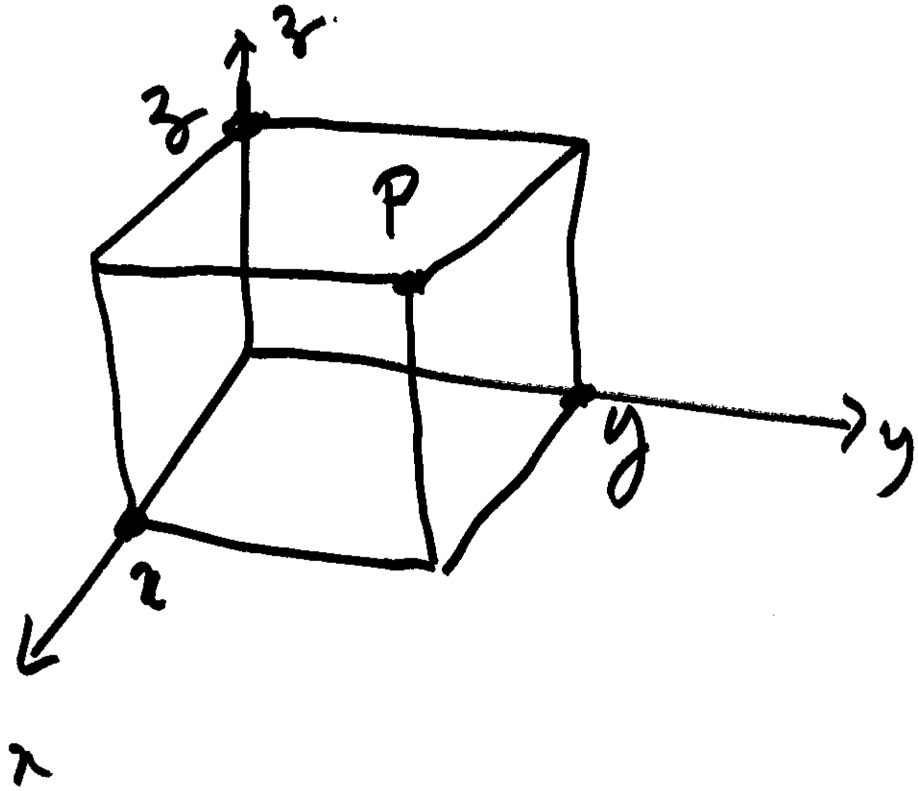
$f\left(\sqrt{\tfrac{3}{2}}, -\sqrt{\tfrac{3}{2}}\right) = f\left(-\sqrt{\tfrac{3}{2}}, \sqrt{\tfrac{3}{2}}\right) = -\tfrac{3}{2}$.

when $x=0$ or $y=0$ $f(x,y)=0$.

Maximum of $f = 3/2$.

Minimum of $f = -3/2$.

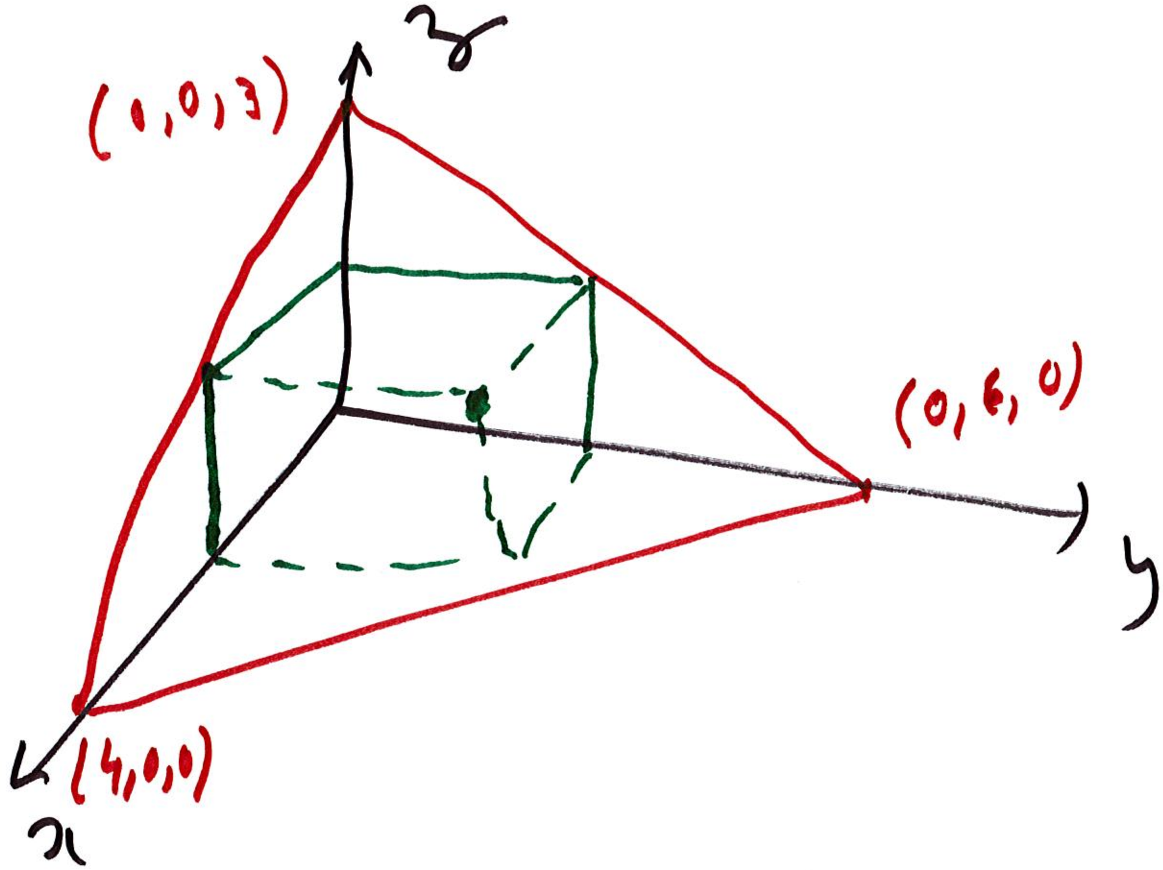
Example II :



We want to build a box as in the picture and we want $P(x, y, z)$ to be on the plane

$$3x + 2y + 4z = 12.$$

Find the maximum ~~value~~ volume of the box.



<u>Solution:</u> $V(x,y,z) = xyz$

$$3x + 2y + 4z = 12$$

<u>Lagrange's Method:</u>

$$g = 3x + 2y + 4z.$$

$$V = xyz.$$

$$\begin{cases} \nabla V = \lambda \nabla g. \\ 3x + 2y + 4z = 12 \end{cases}$$

$$\nabla V = \langle yz, xz, xy \rangle$$

$$\nabla g = \langle 3, 2, 4 \rangle$$

$$\begin{cases} \langle yz, xz, xy \rangle = \lambda \langle 3, 2, 4 \rangle \\ 3x + 2y + 4z = 12 \end{cases}$$

$$\begin{cases} yz = 3\lambda \\ xz = 2\lambda \\ xy = 4\lambda \\ 3x + 2y + 4z = 12 \end{cases}$$

$x \neq 0, \ y \neq 0, \ z \neq 0$

Divide the first two equations by each other

$$\frac{yz}{xz} = \frac{3\lambda}{2\lambda} \ ; \ \boxed{y/x = 3/2}$$

Divide the 1st equation by the 3rd:

$$\frac{\cancel{y}z}{x\cancel{y}} = \frac{3\cancel{\lambda}}{4\cancel{\lambda}} \quad z/x = 3/4.$$

$$\boxed{y = \tfrac{3}{2}x} \quad z = \tfrac{3}{4}x.$$