Advisory Letter Grades

A - 92

B^+ - 84

B^- - 76

C - 68

C^- - 60

D - 52

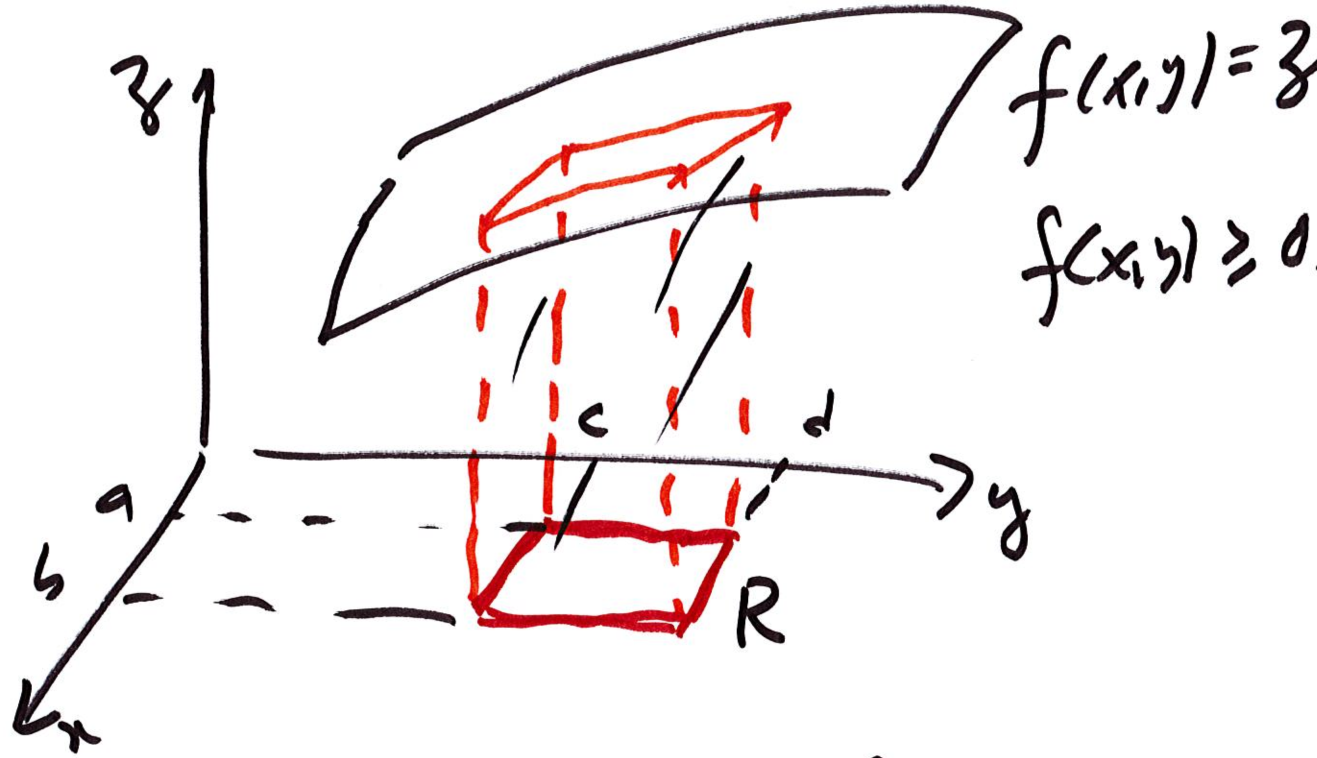
F - 44

Average ~ 68

Median ~ 68

Lesson 19 Section 15.1

Multiple and Iterated Integrals



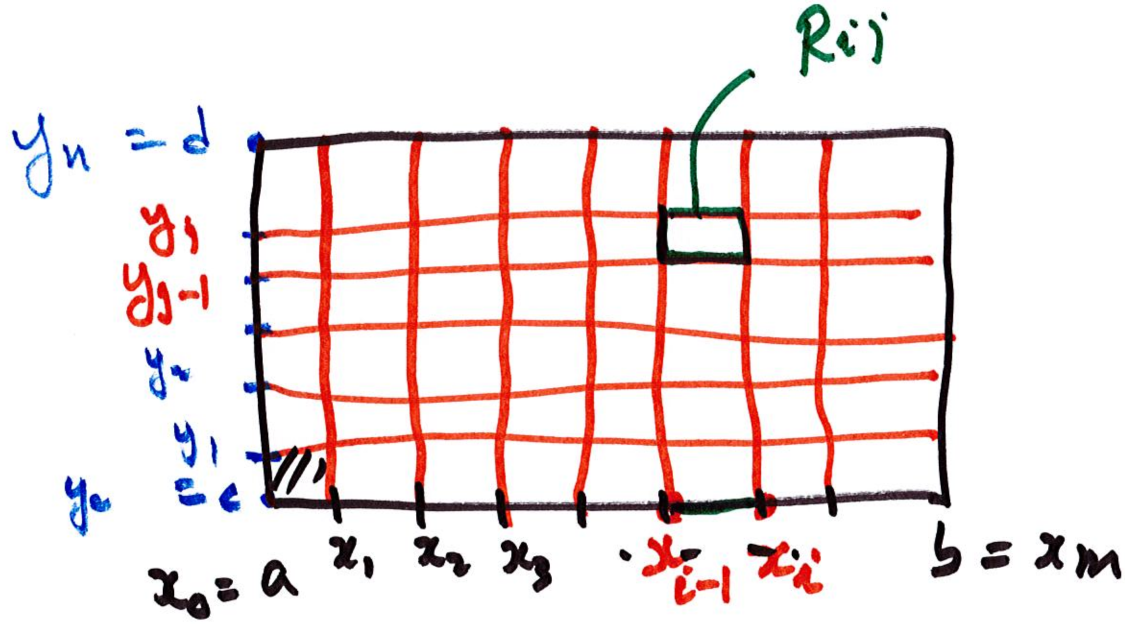
R is a rectangle $R=\{a\leq x\leq b$
$c\leq y\leq d\}$

Find the volume of the solid above R and below the graph of $f(x,y)$.

Step 1 : Divide

$[a,b]$ into m equal parts

$[c,d]$ into n equal parts.



We have $m \times n$ rectangles.

Area of each rectangle $= A = \frac{b-a}{m} \cdot \frac{d-c}{n}$

Step 2 : In each rectangle R_{ij}

pick a point (x_{ij}^*, y_{ij}^*)

Step 3: Form the Sum:

$$f(x_{11}^*, y_{11}^*) \cdot A + f(x_{11}^*, y_{12}^*) A$$

$+ \dots$ Sum over all rectangles.

$$\sum_{\substack{0 \le i \le m \\ 0 \le j \le n}} f(x_{ij}^*, y_{ij}^*) A$$

Step 4:

$$\iint_R f(x,y)\, dA = \lim_{\substack{m \to \infty \\ n \to \infty}} \sum_{\substack{0 \le i \le m \\ 0 \le j \le n}} f(x_{ij}^*, y_{ij}^*) A$$

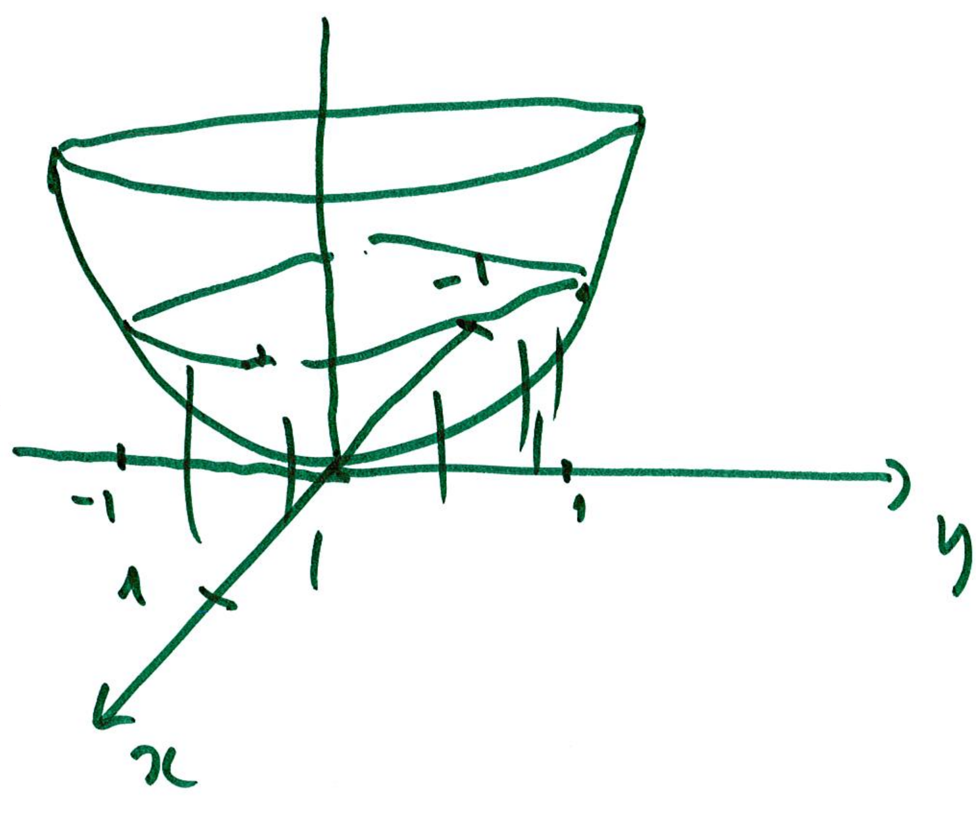
Example: Approximate the value of the volume under the graph of $f(x,y) = x^2+y^2$

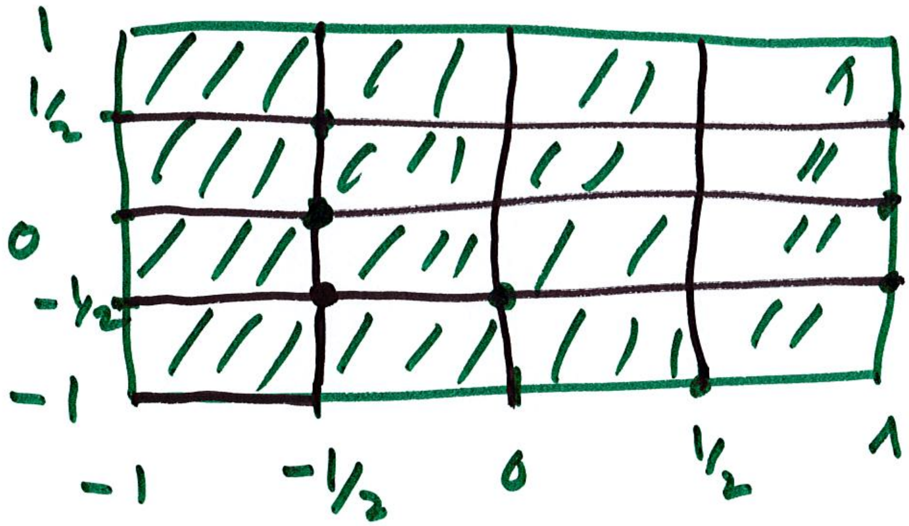
above $R = \{-1 \le x \le 1, -1 \le y \le 1\}$

by taking $m=4$, $n=4$ and

(x_{ij}^*, y_{ij}^*) = upper corner

of each rectangle.





Area of each rectangle $= \dfrac{1}{4}$

$$\sum_{\substack{0 \le i \le 4 \\ 0 \le j \le 4}} f(x_{ij}^*, y_{ij}^*)\ 1/4 =$$

$f(x,y) = x^2 + y^2.$

$$= f(-1/2, -1/2) \cdot 1/4 + f(-1/2, 0) \cdot 1/4$$
$$+ f(-1/2, 1/2)\ 1/4 + f(1/2, 1) \cdot 1/4$$
$$+ f(0, -1/2) \cdot 1/4 + f(0,0) \cdot 1/4 + f(0, 1/2)\ 1/4$$
$$+ f(1/2, -1/2) \cdot 1/4 + f(1/2, 0)\ 1/4$$
$$+ f(0,1)\ 1/4$$

$$+ f(1/2, 1/2)\,1/4 + f(1/2, 1)\,1/4$$

$$+ f(1, -1/2)\,1/4 + f(1, 0)\,1/4$$

$$+ f(1, 1/2)\,1/4 + f(1, 1)\,1/4.$$

$$= \frac{1}{4}\Big(1/2 + 1/4 + 1/2 + 5/4 + 1/4$$

$$+ 0 + 1/4 + 1 + 1/2 + 1/4$$

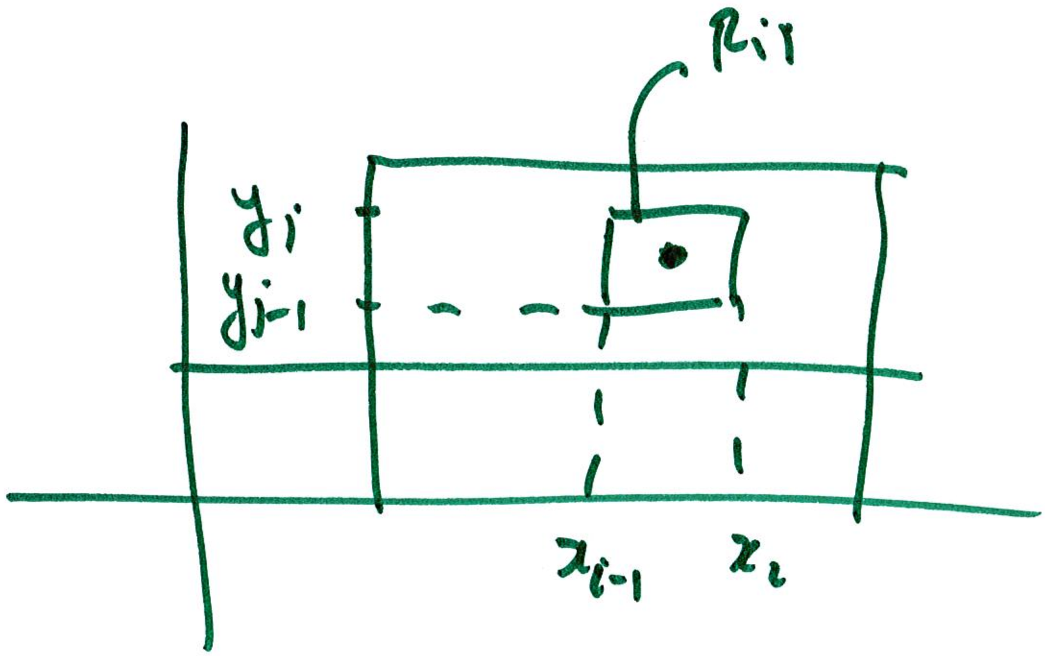
$$+ 1/2 + 5/4 + 5/4 + 1 + 5/4 + 2\Big)$$

$$= 3$$

check this.

The midpoint Rule

Pick $(x_{ij}^*, y_{ij}^*) =$ center of the rectangle R_{ij}



$$x_{ij}^* = \frac{x_i + x_{i-1}}{2}$$

$$y_{ij}^* = \frac{y_j + y_{j-1}}{2}.$$

Iterated Integrals

$$R = \{ a \le x \le b,\ c \le y \le d \}$$

$$\int_c^d \Big(\underbrace{\int_a^b f(x,y)\, dx}_{} \Big)\, dy$$

Fix y and integrate in x.

Example: Compute

$$\int_0^1 \Big(\int_0^1 x y^3 \, dx \Big)\, dy$$

$$= \int_0^1 \Big(y^3 \overbrace{\int_0^1 x\, dx}^{=1/2} \Big)\, dy$$

$$= \int_0^1 \frac{1}{2} y^3 \, dy = \frac{1}{2} \cdot \frac{1}{4} y^4 \Big|_0^1$$

$$= 1/8.$$

$$\int_0^1 \left(\underbrace{\int_0^1 x \sin(xy)\, dy} \right) dx$$

$$\int_0^1 x \sin(xy)\, dy = +x \int_0^1 \sin(xy)\, dy$$

$$= x \; -\frac{1}{x}\cos(xy) \Big|_0^1 = -\cos(xy)\Big|_0^1$$

$$= -\cos x + 1 = 1 - \cos x.$$

$$\int_0^1 \left(\int_0^1 x \sin(xy)\, dy \right) dx$$

$$= \int_0^1 (1 - \cos x)\, dx = 1 - \sin x \Big|_0^1$$

$$= \underline{1 - \sin 1}.$$

If $f(x,y)$ is a continuous function on $a \le x \le b$

$c \le y \le d$

$$\iint_R f(x,y)\,dA = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

$$= \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy.$$

Find the volume under the graph of $f(x,y) = x^2 + y^3$ above the rectangle

$0 \le x \le 1, \quad 0 \le y \le 2.$

$$\iint_R (x^2 + y^3)\, dA = \int_0^2 \left(\int_0^1 (x^2 + y^3)\, dx \right) dy$$

$$= \int_0^2 \left(\tfrac{1}{3} + y^3 \right) dy = \tfrac{1}{3} \cdot 2 + \frac{y^4}{4} \Big|_0^2$$

$$= \tfrac{2}{3} + 4.$$

$$\int_0^1 (x^2 + y^3)\, dx = \frac{x^3}{3} + y^3 \cdot x \Big|_0^1 = \tfrac{1}{3} + y^3.$$