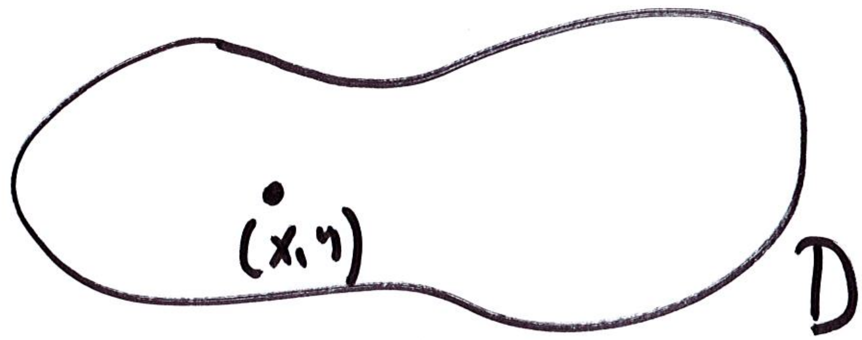
Lesson 26   Section 16.1

Vector Fields

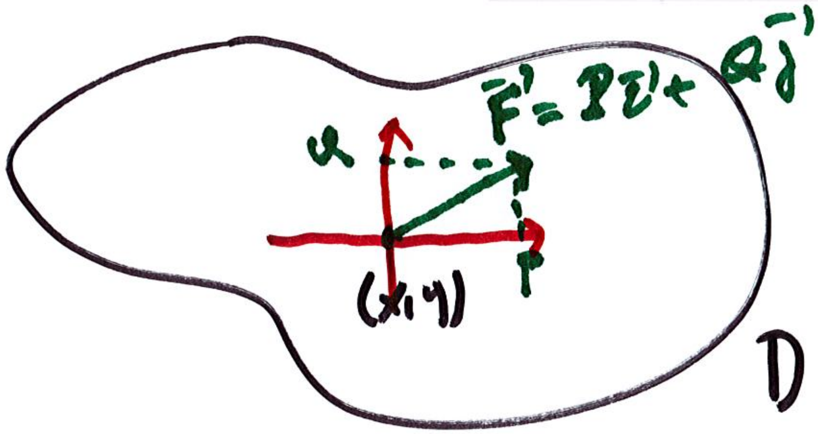
2-dimensions



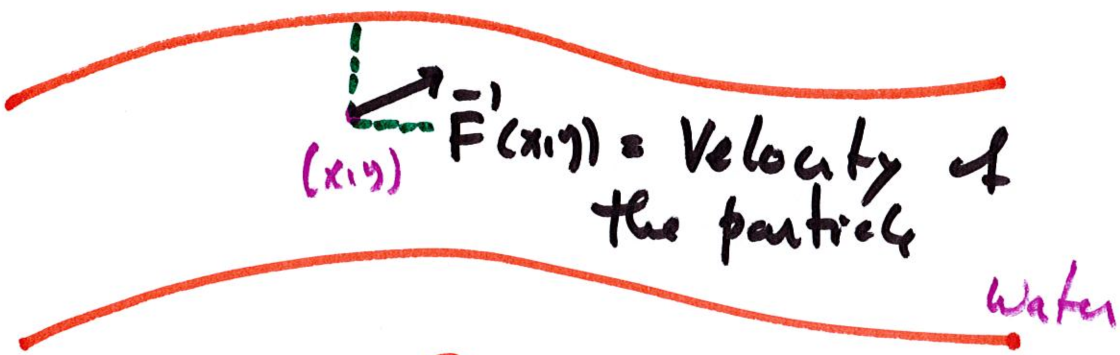
A vector field is a function

$$\vec{F}(x,y) = P(x,y)\,\vec{i} + Q(x,y)\,\vec{j}.$$

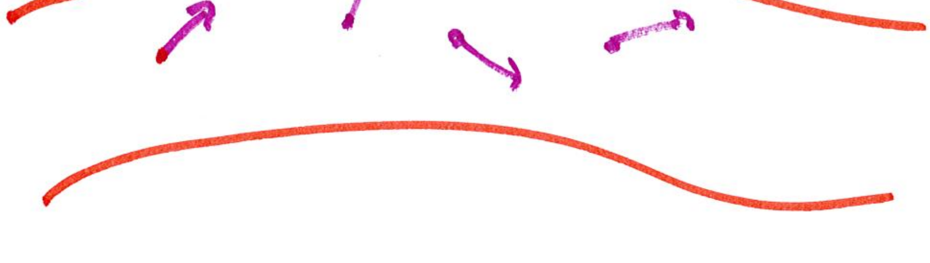
Sketch the "graph" of the vector field $\vec{F}$.



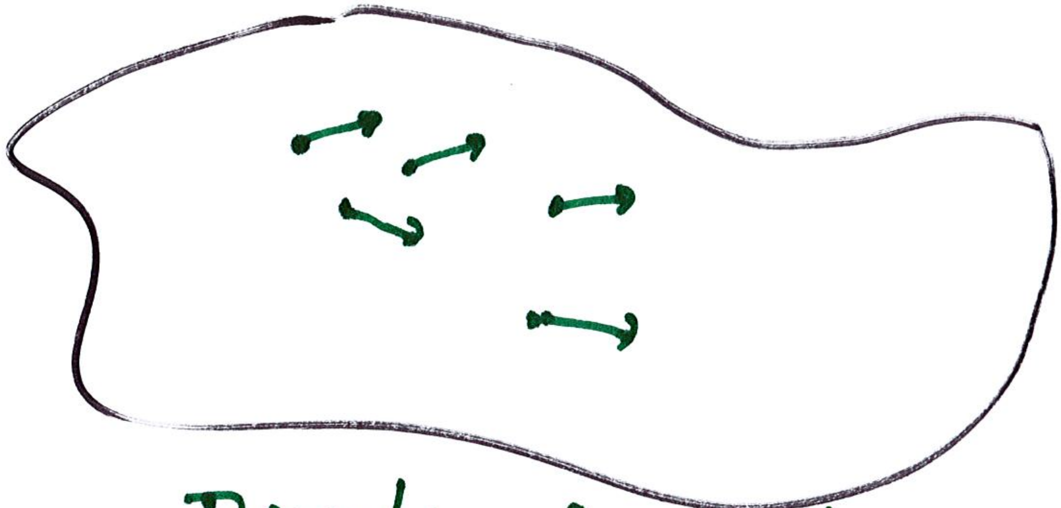
Examples. Motion of Fluids



River.

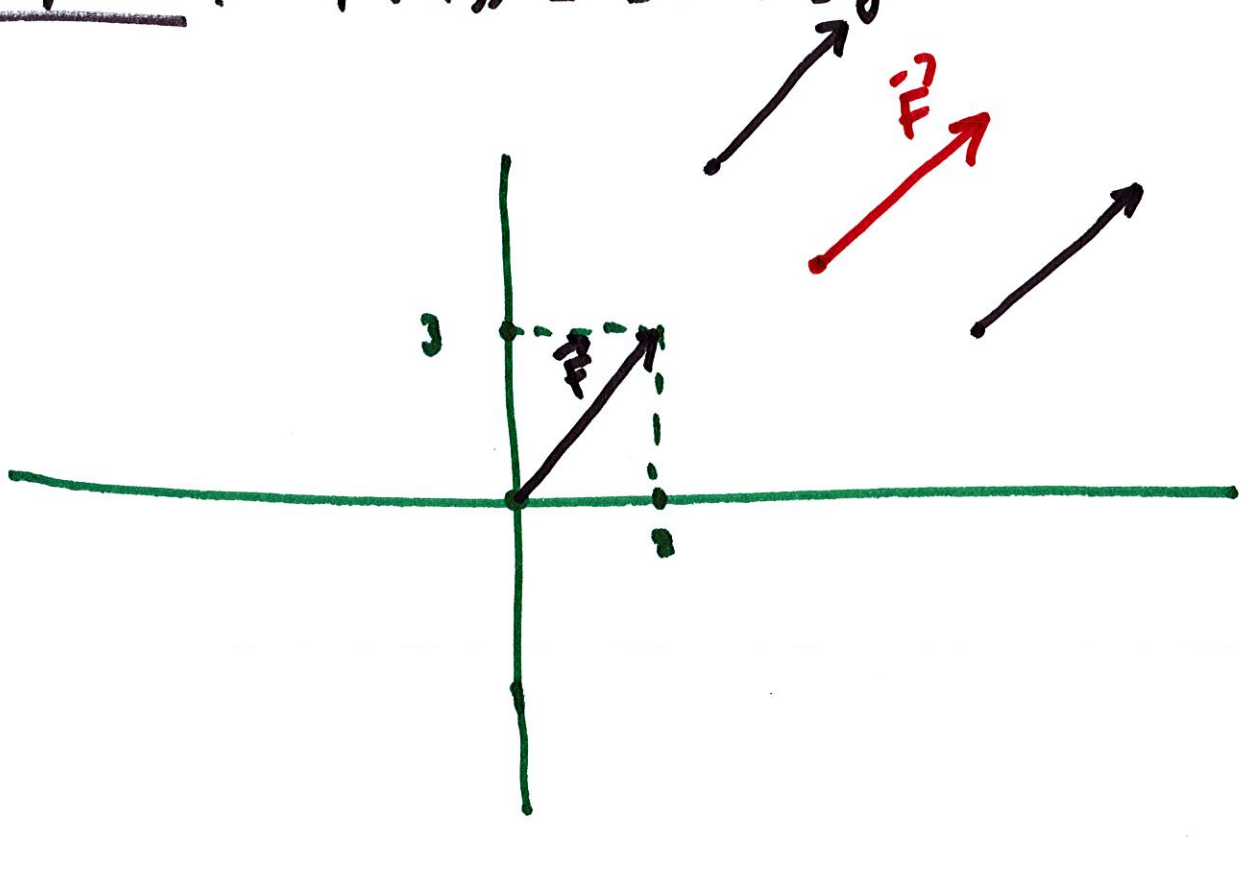


Weather:

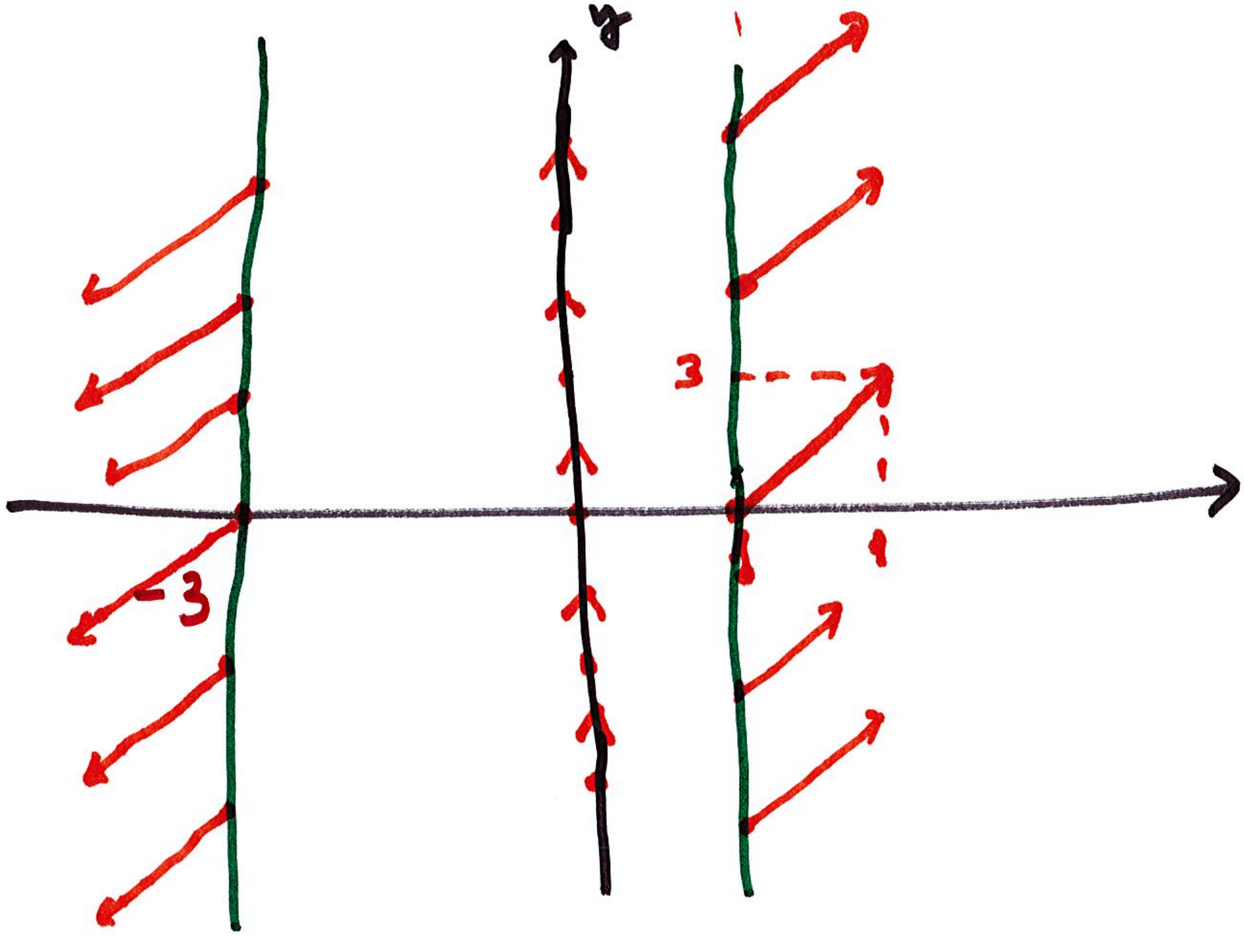


Direction of Wind

Examples: $\vec{F}(x,y) = 2\vec{i} + 3\vec{j}$



$$\vec{F}(x,y) = \langle x, x+2 \rangle = x\vec{i} + (x+2)\vec{j}$$



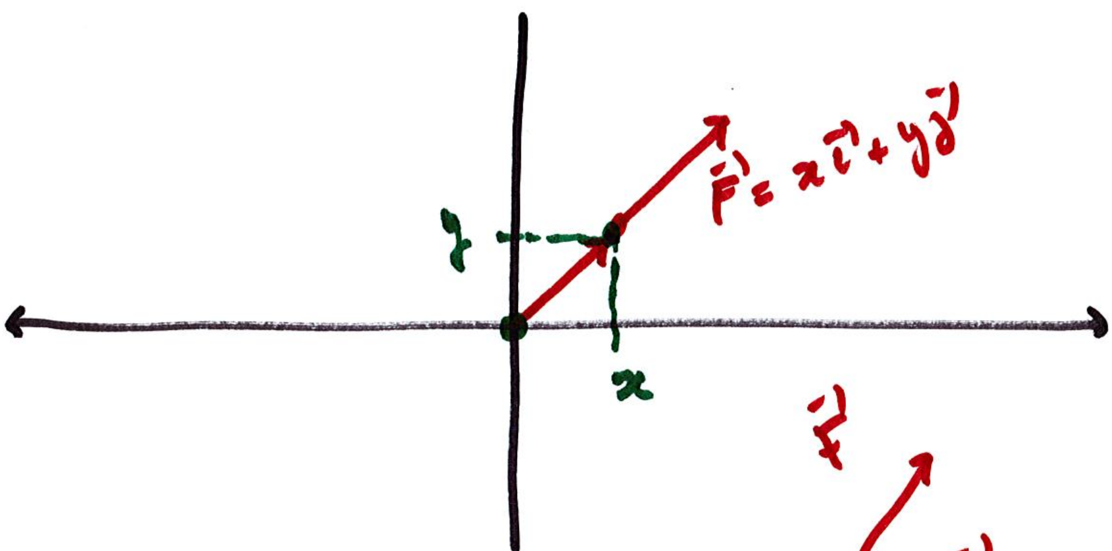
$$\vec{F}(0,y) = 0\vec{i} + 2\vec{j} = 2\vec{j}$$

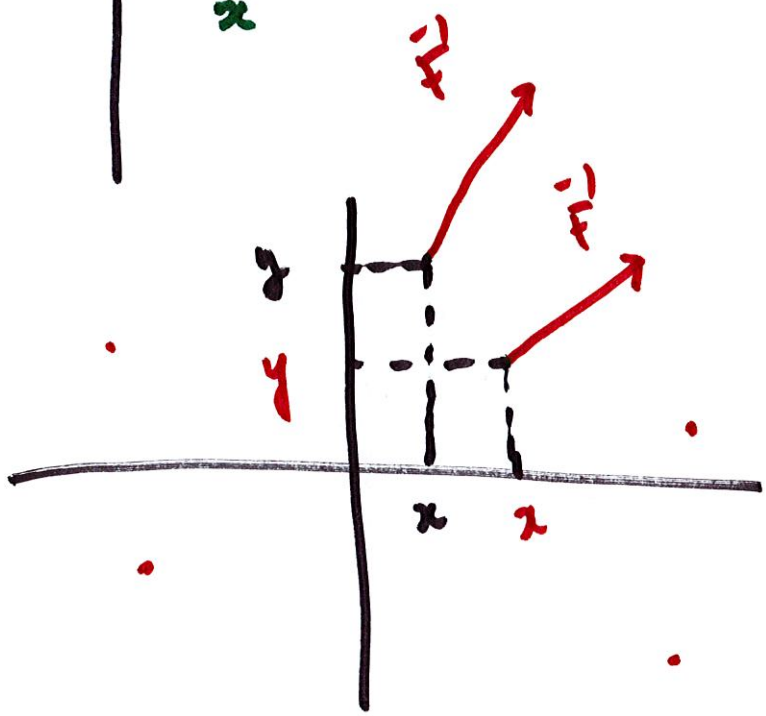
$$\vec{F}(1,y) = \vec{i} + 3\vec{j}.$$

$$\vec{F}(-3,y) = -3\vec{i} - \vec{j}.$$

The Radial vector field

$$\vec{F}(x,y) = x\vec{i} + y\vec{j}.$$

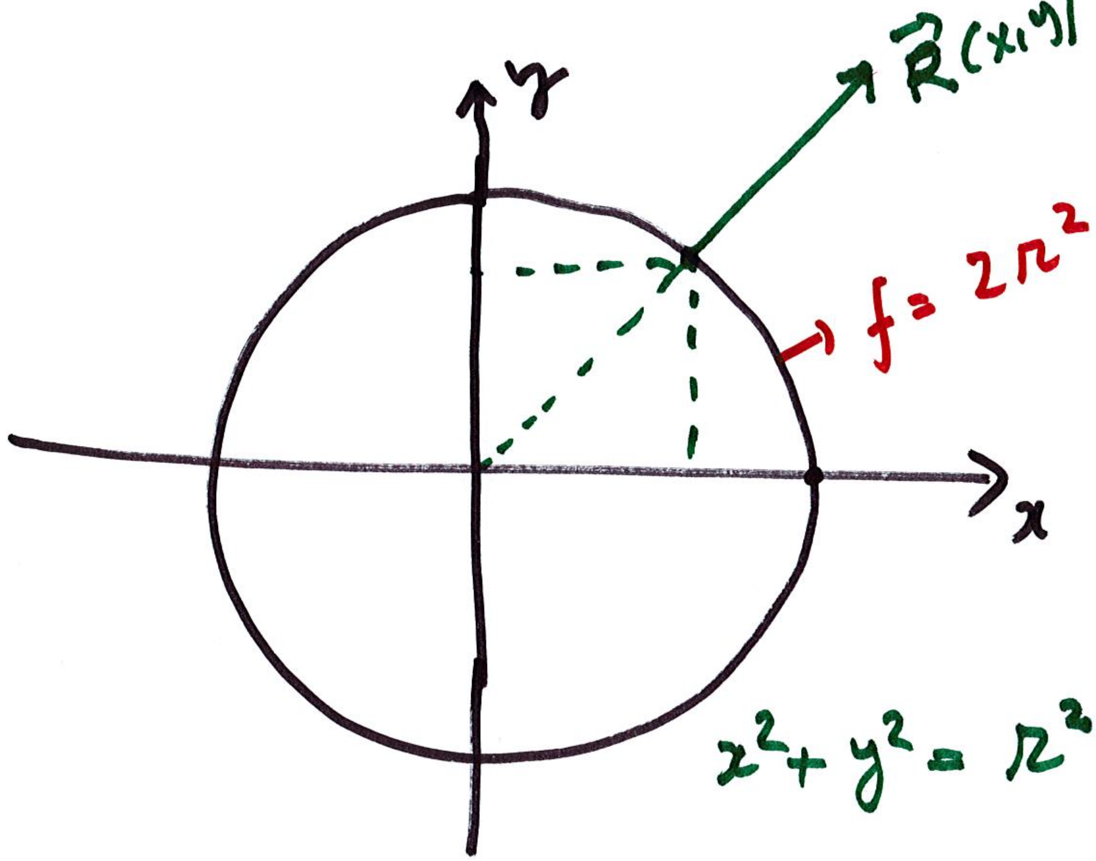




$\vec{F}(0,0) = 0$

$$\boxed{\vec{F}(x,y) = -y\vec{i} + x\vec{j}.}$$

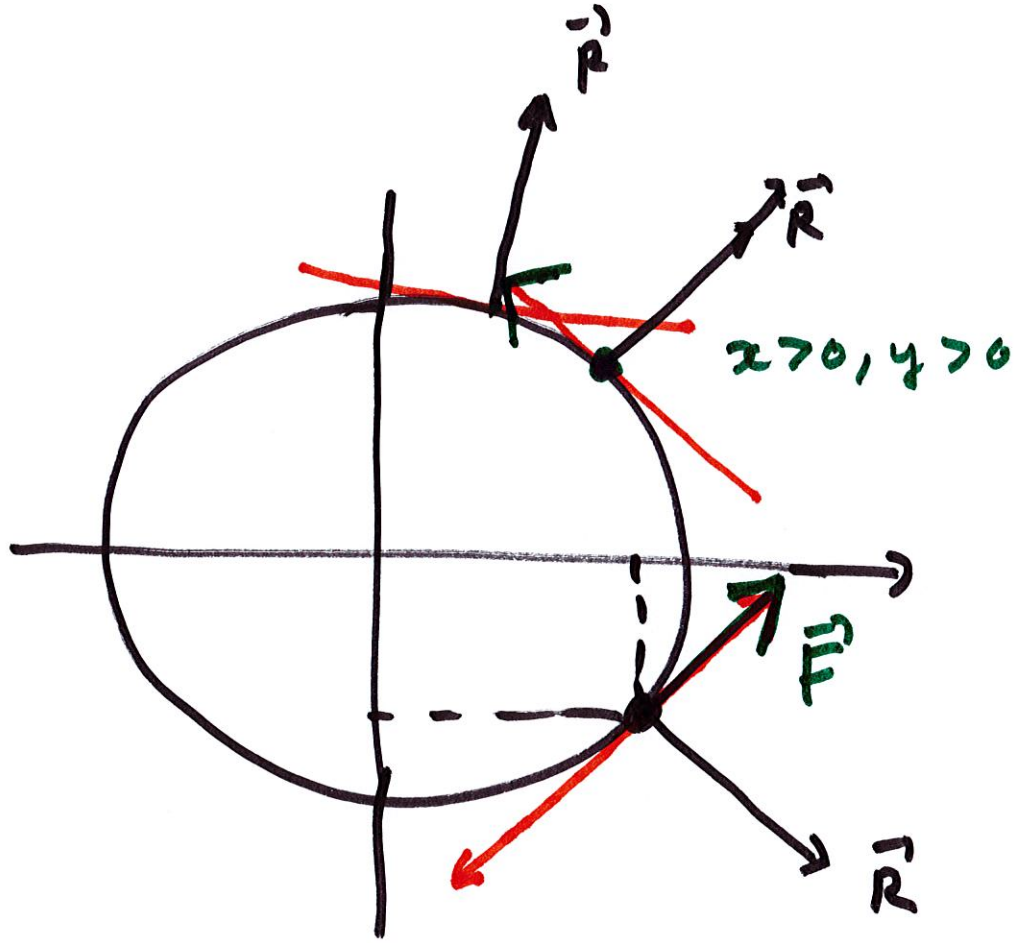
$$\vec{R}(x,y) = x\vec{i} + y\vec{j}$$



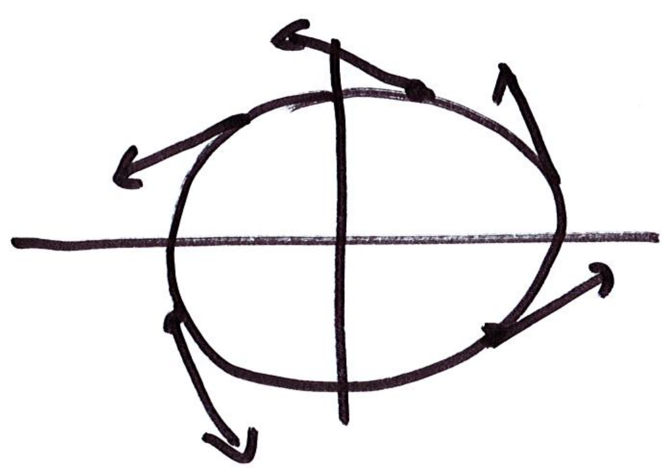
$$f(x,y) = \frac{1}{2}(x^2 + y^2)$$

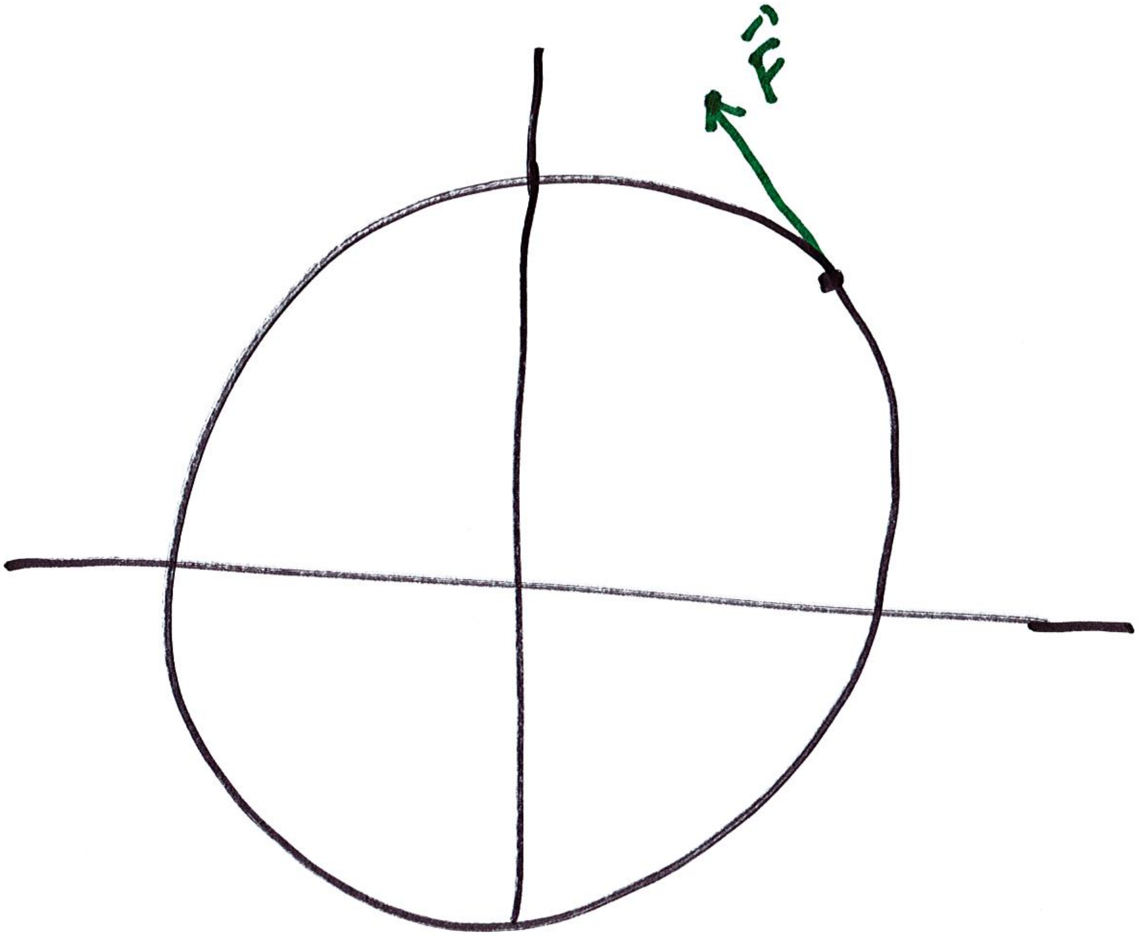
$$\boxed{\vec{\nabla} f(x,y) = x\vec{i} + y\vec{j} = \vec{R}(x,y)}$$

$$\vec{F} \cdot \vec{R} = -\underline{y}x + xy = 0$$



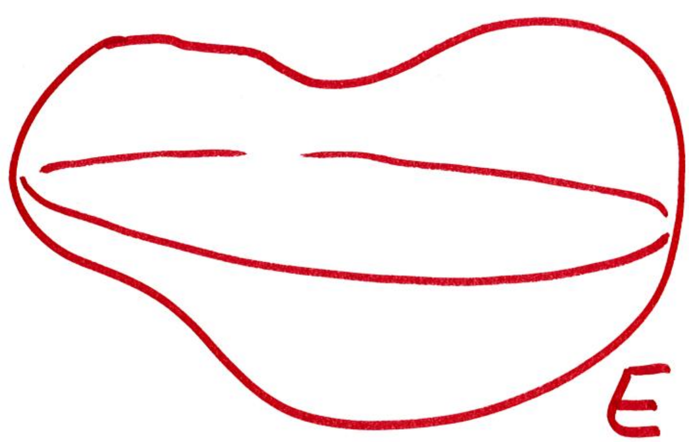
$$\vec{F}(x,y) = -\underline{y}\,\vec{i} + \underline{x}\,\vec{j}$$





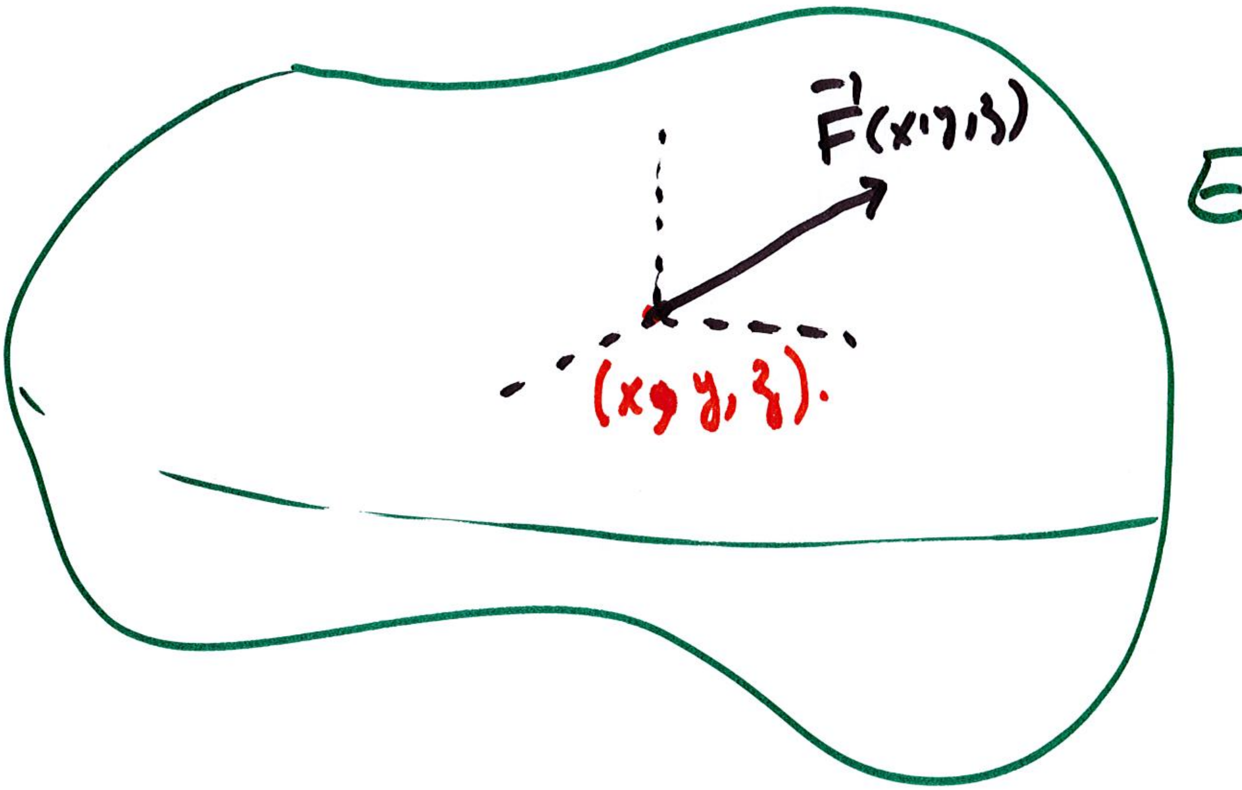
3- Dimensions

Region E in $\mathbb{R}^3$.

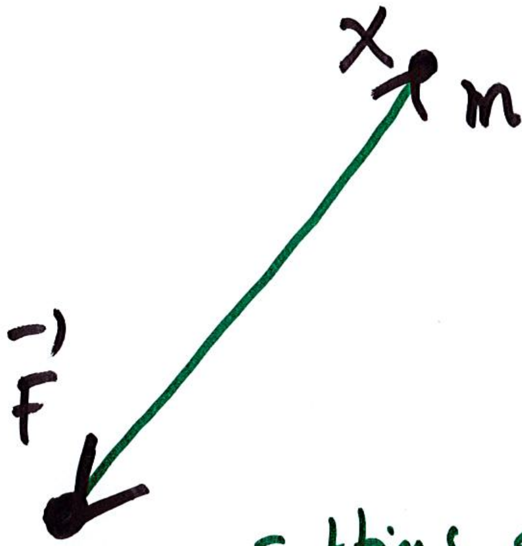


$$\vec{F}(x,y,z) = P(x,y,z)\,\vec{i} + Q(x,y,z)\,\vec{j} + R(x,y,z)\,\vec{k}$$

(x,y,z) in E.



Newton's Law of Gravitation



$\vec{X} = x\vec{i} + y\vec{j} + z\vec{k}$

M = sitting at the origin.

$$|\vec{F}| = \frac{mMG}{|\vec{x}|^2}$$

The direction of $\vec{F}$ is

$$-\frac{\vec{x}}{|\vec{x}|}$$

$$\boxed{\vec{F} = -\frac{mMG}{|\vec{x}|^3}\,\vec{x}}$$

Gradient Vector fields

$$\vec{F}(x,y,z) = \vec{\nabla} f(x,y,z)$$

$$= \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}.$$

Example: $f(x,y,z) = xyz + z^3$

$$\vec{\nabla} f = yz\,\vec{i} + xz\,\vec{j} + (xy + 3z^2)\,\vec{k}$$

Question: Given $\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$

Is $\vec{F}$ the gradient vector field of some function f?

$$F(x,y,z) = yz\,\vec{i} + xz\,\vec{j} + (xy + 3z^2)\,\vec{k}.$$

Find $f(x,y,z)$

$$\frac{\partial f}{\partial x} = yz, \quad \frac{\partial f}{\partial y} = xz$$

$$\frac{\partial f}{\partial z} = xy + 3z^2.$$

$$\frac{\partial f}{\partial x} = yz \; ; \quad f(x,y,z) = yzx + W(y,z)$$

$$\frac{\partial f}{\partial y} = xz + \frac{\partial W}{\partial y} = xz$$

$$\frac{\partial W}{\partial y} = 0.$$