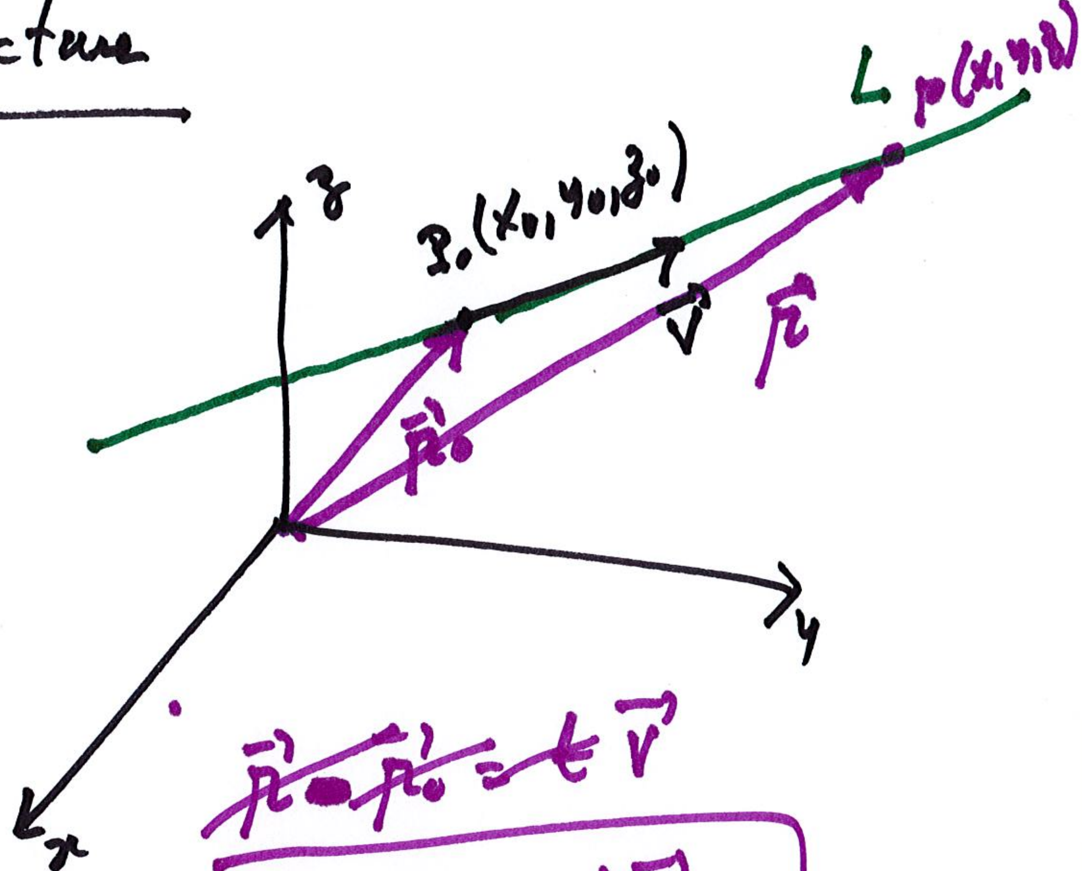
Lesson 3 Sections 12.5 and 12.6

Last Lecture

Lines:



$$\cancel{\vec{r} - \vec{r_0} = t\vec{v}}$$

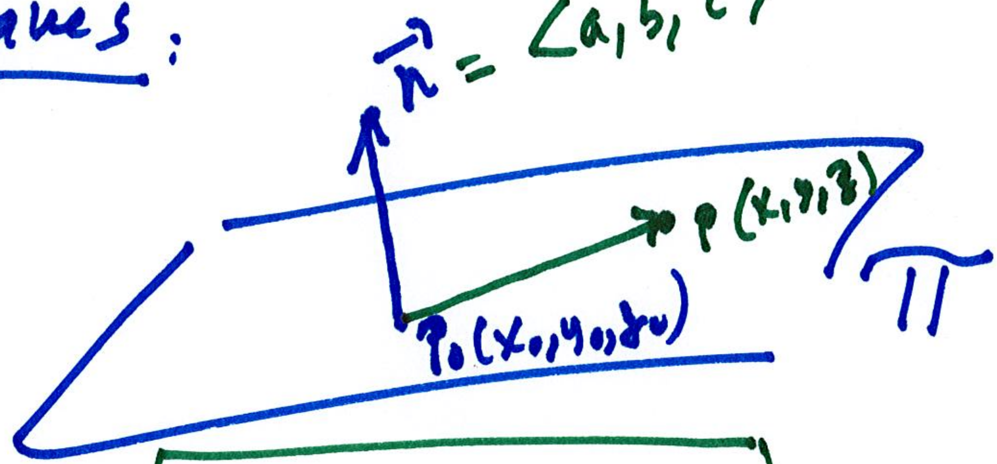
$$\boxed{\vec{r} - \vec{r_0} = t\vec{v}}$$

In coordinates: $\vec{r} - \vec{r_0} = \langle x - x_0, y - y_0, z - z_0 \rangle$

$\vec{v} = \langle a, b, c \rangle$

$$\boxed{x - x_0 = ta, \quad y - y_0 = tb, \quad z - z_0 = tc}$$

Planes:



$$\boxed{\overrightarrow{P_0P} \cdot \vec{n} = 0}$$

$$\overrightarrow{P_0P} = \langle x - x_0,\ y - y_0,\ z - z_0 \rangle$$

$$\overrightarrow{P_0P} \cdot \vec{n} = a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

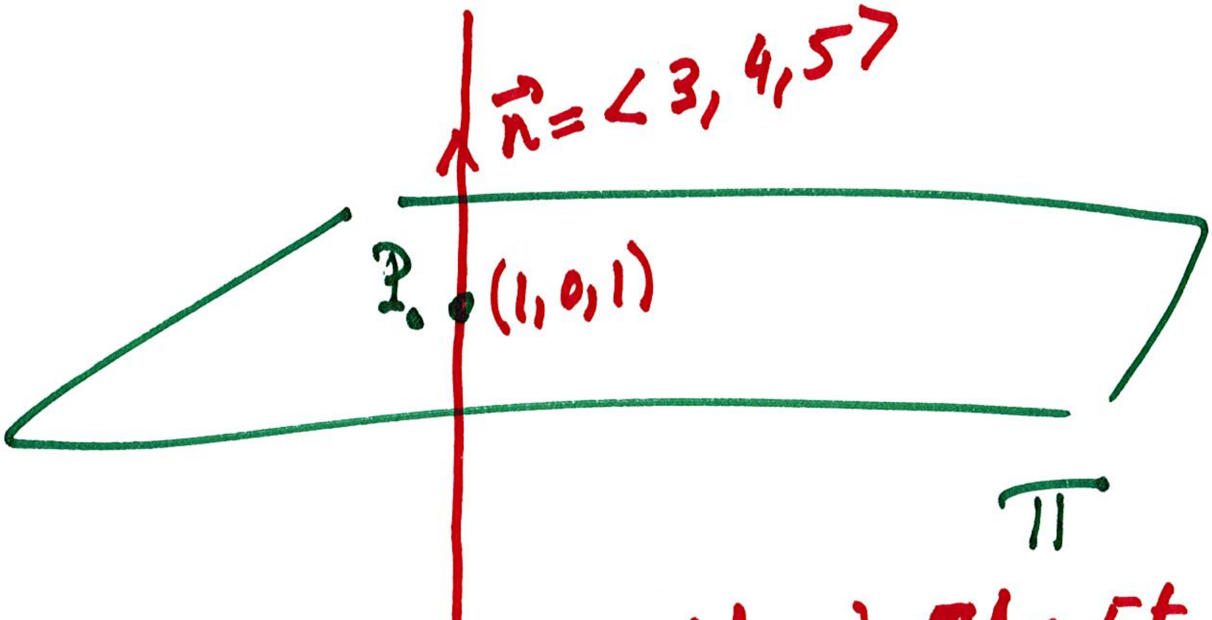
Remark: a, b, c are the coefficients of ~~the~~ x, y, z respectively.

Ex: $3x + 2y + 4z = 5$.

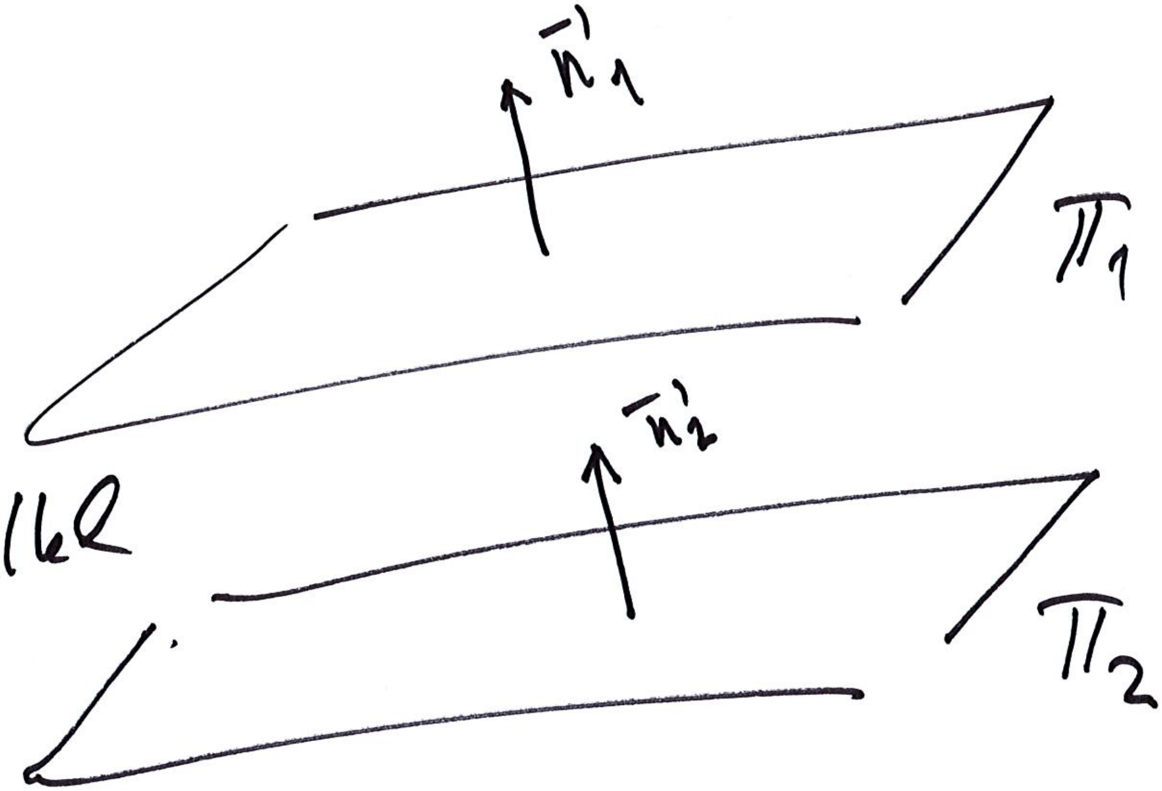
$\vec{n} = \langle 3, 2, 4 \rangle$

Find a parametric equation of the line which passes $P_0(1, 0, 1)$ and is perpendicular to the plane

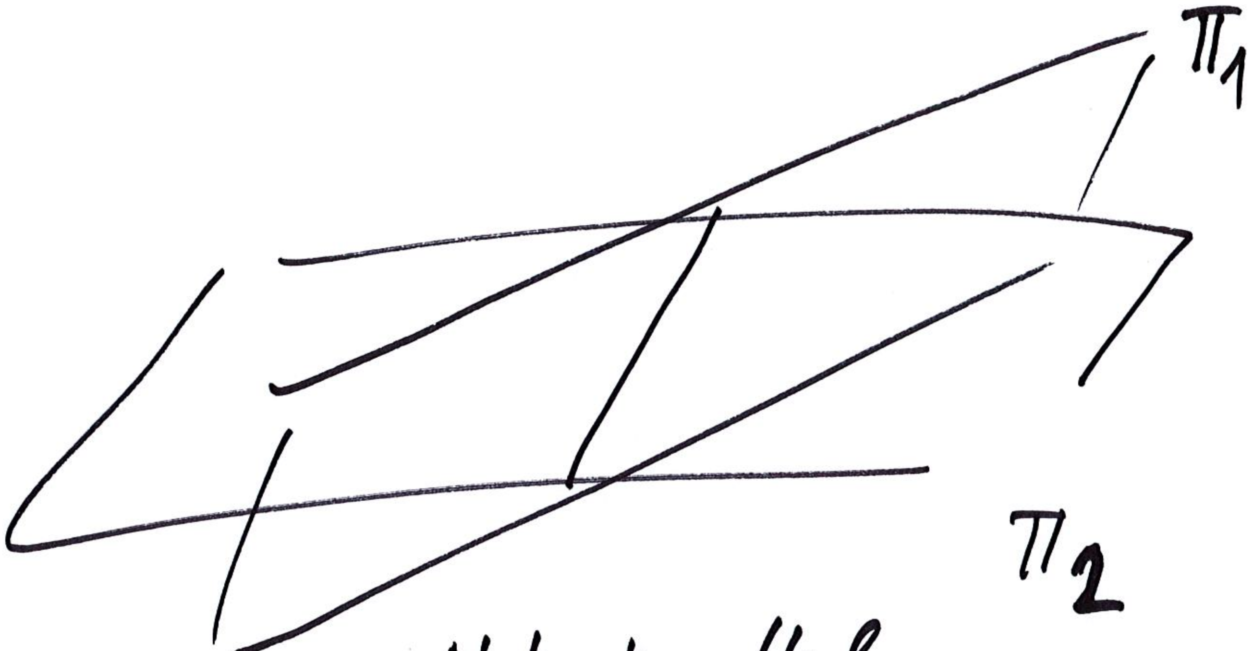
$$3x + 4y + 5z = 8$$



$$x - 1 = t3 \; ; \quad y - 0 = 4t, \quad z - 1 = 5t.$$



Parallel



Not parallel.

Π_1 and Π_2 are parallel if their normal vectors are parallel.

$$\vec{n}_1 = t\,\vec{n}_2.$$

$$x-1=3t, \quad y=4t, \quad z-1=5t$$

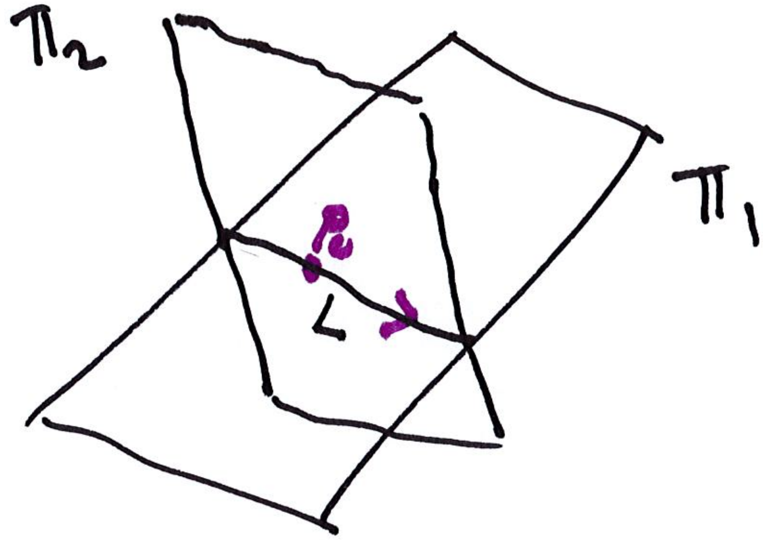
$$t=\frac{x-1}{3}=\frac{y}{4}=\frac{z-1}{5}.$$

$$\boxed{\frac{x-1}{3}=\frac{y}{4}=\frac{z-1}{5}}$$

Symmetric Equations

Example 2: Find ~~an~~ parametric equation for the line of intersection of the planes

$$\begin{cases} 3x+2y+z=5 & \vec{n}_1=\langle 3,2,1\rangle \\ x+y-z=8 & \vec{n}_2=\langle 1,1,-1\rangle \end{cases}$$



Need to find a point $P_0(x_0, y_0, z_0)$ on the line and a vector $\vec{v}$ which is parallel to the line.

Let's first find $\vec{v}$.

$\vec{v}$ lies on Π_1 (parallel to Π_1)

$\vec{v}$ lies on Π_2

$\vec{v}$ is perpendicular to $\underline{\underline{\vec{n_1}}}$ and $\underline{\underline{\vec{n_2}}}$

One choice of $\vec{v}$ is

$$\vec{v} = \vec{n_1} \times \vec{n_2} \,.$$

$\vec{n_1} = \langle 3, 2, 1 \rangle$, $\vec{n_2} = \langle 1, 1, -1 \rangle$

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \vec{i}(-3) - \vec{j}(-4) + \vec{k}(1)$$

$$= -3\vec{i} + 4\vec{j} + \vec{k}.$$

$$\boxed{\vec{v} = \langle -3, 4, 1 \rangle}$$

Now we have to find P_0.

$$\begin{cases} 3x + 2y + z = 5 \\ x + y - z = 8 \end{cases}$$

$\underline{x = 0}$

$$2y + z = 5$$
$$y - z = 8$$

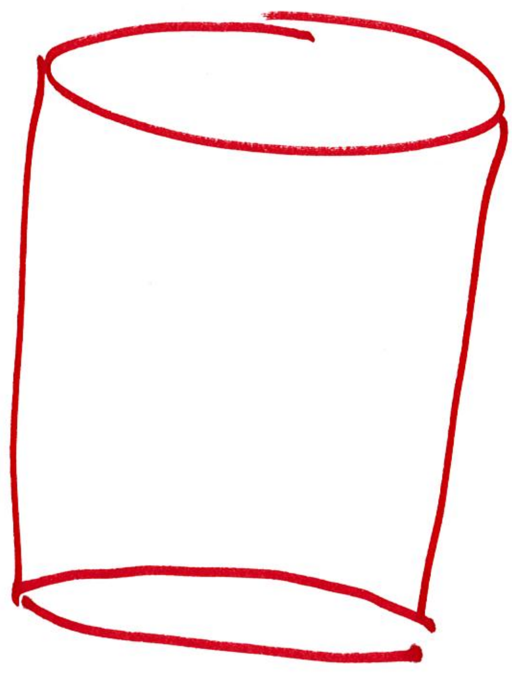
$$3y = 13, \quad y = \frac{13}{3}$$

$$z = y - 8 = \frac{13}{3} - 8 = -\frac{11}{3}$$

$$\boxed{P_0\left(0, \frac{13}{3}, -\frac{11}{3}\right)}$$

Section 12.6 Cylinders and Quadrics.

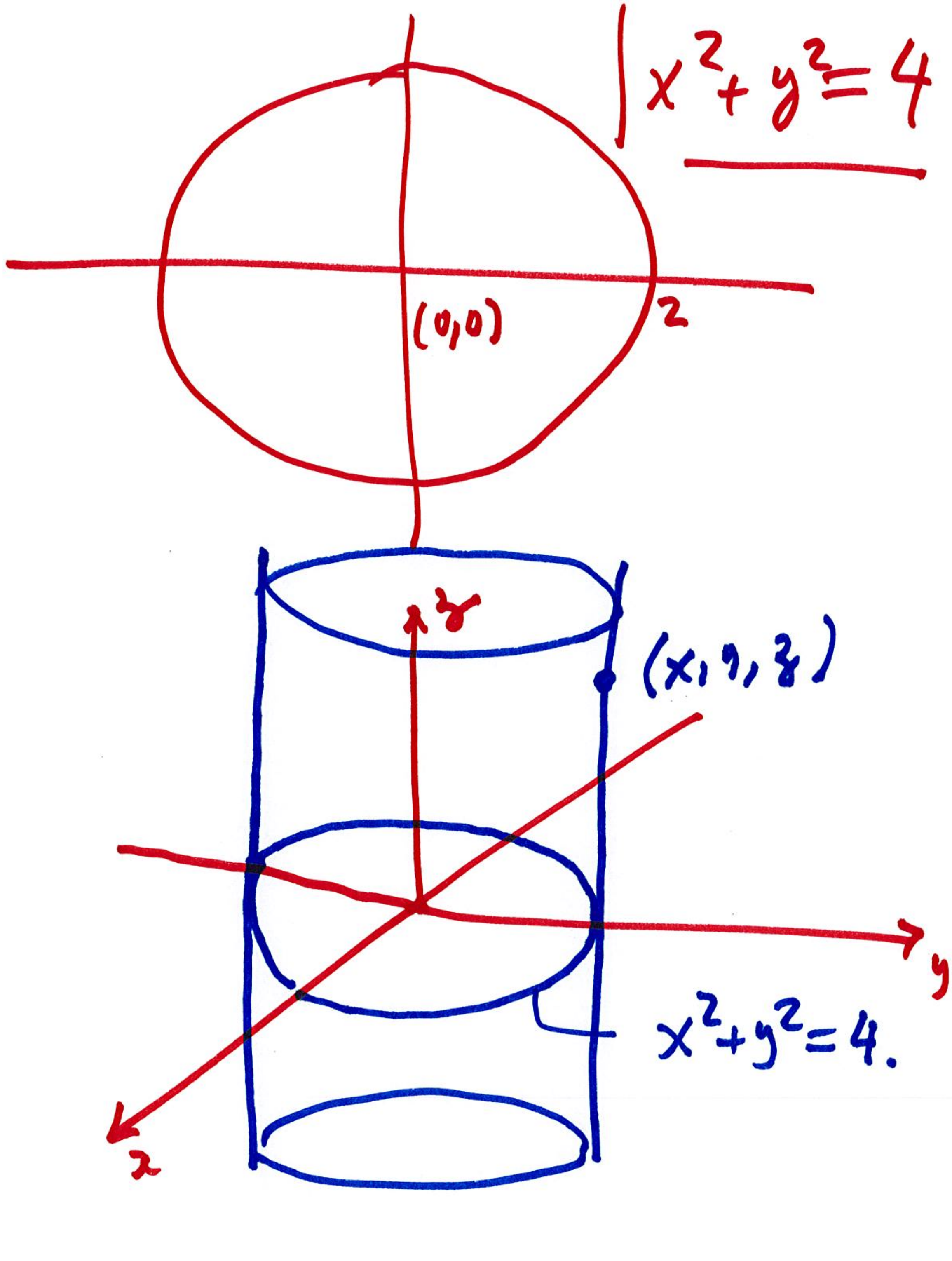
Cylinders



Start with an equation

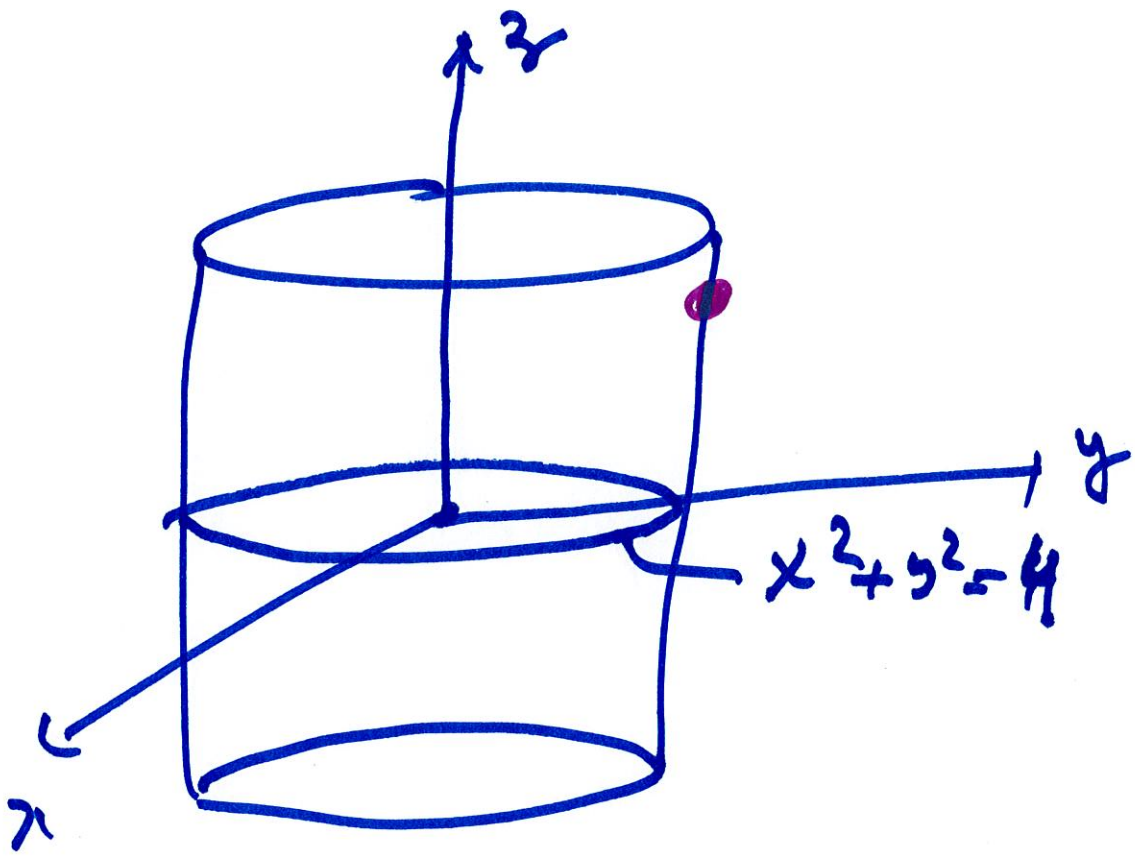
in 2 dimensions

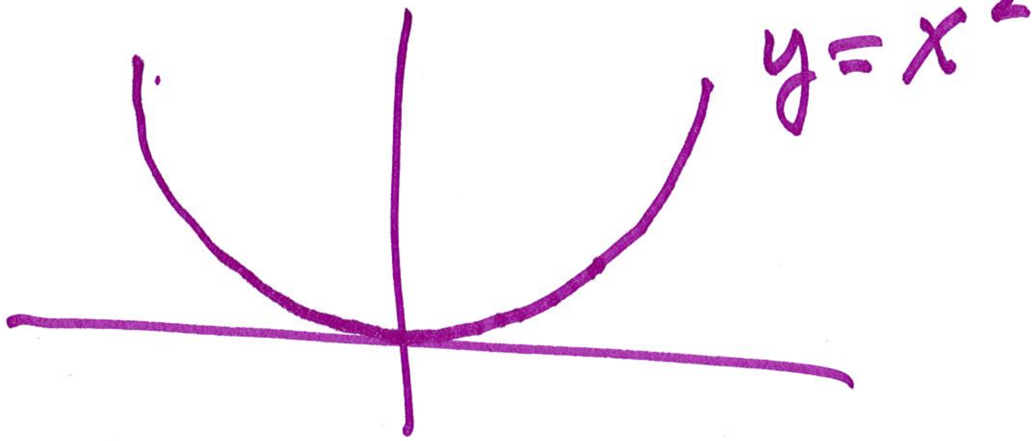
$$\boxed{x^2 + y^2 = 4}$$



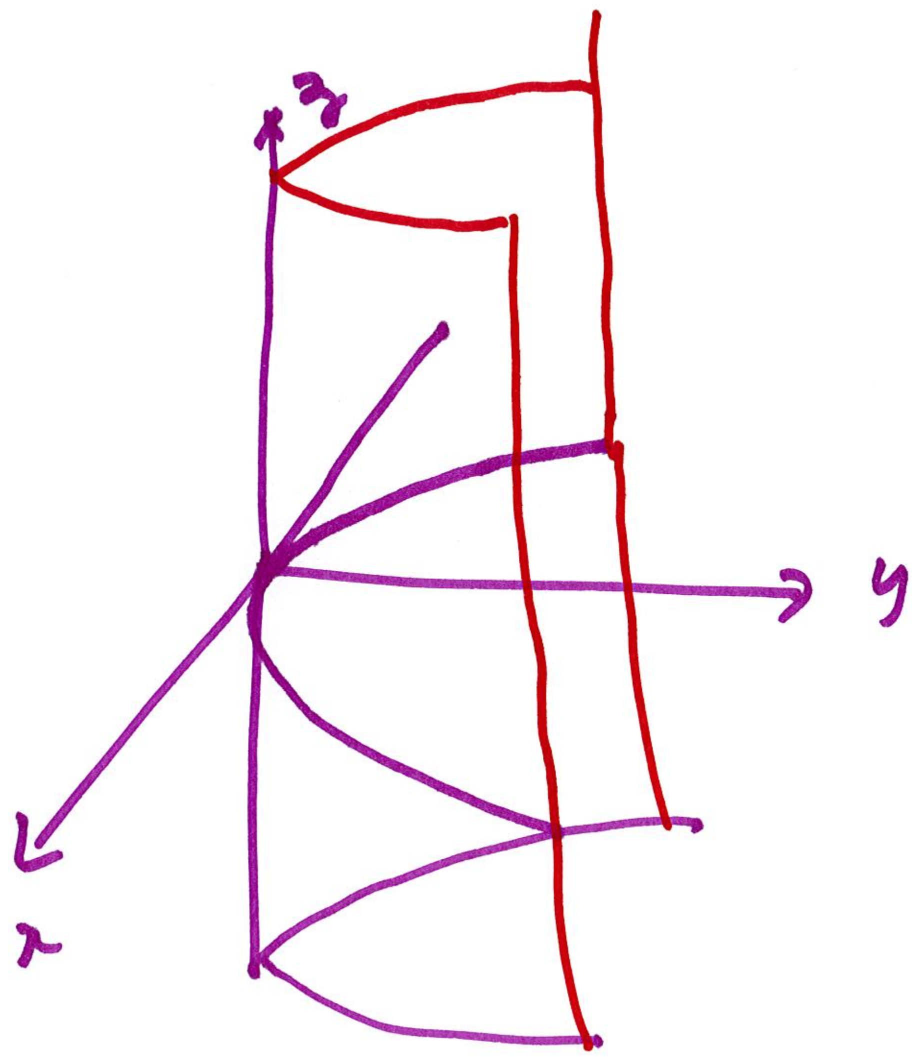
So $x^2+y^2=4$ in 3D

represents a cylinder



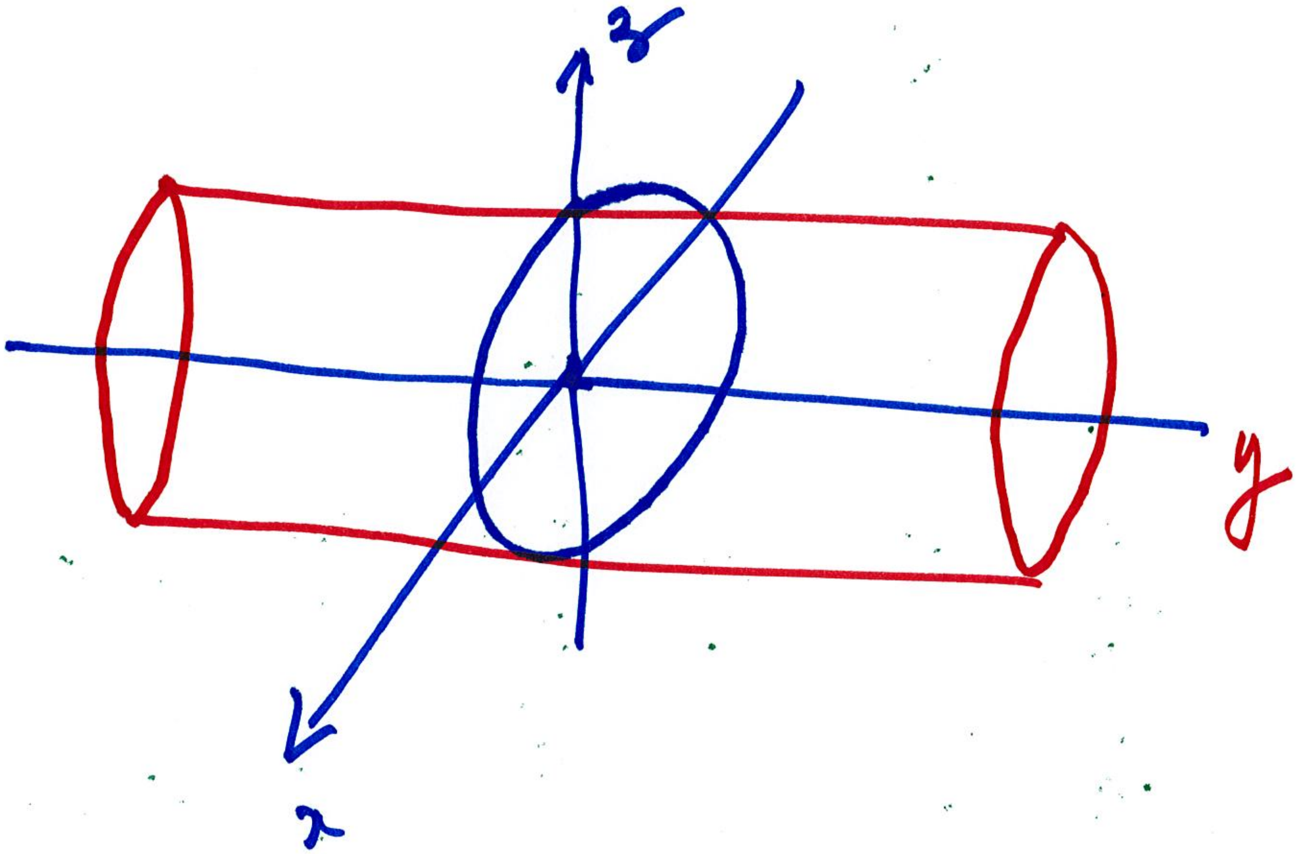


y=x^2



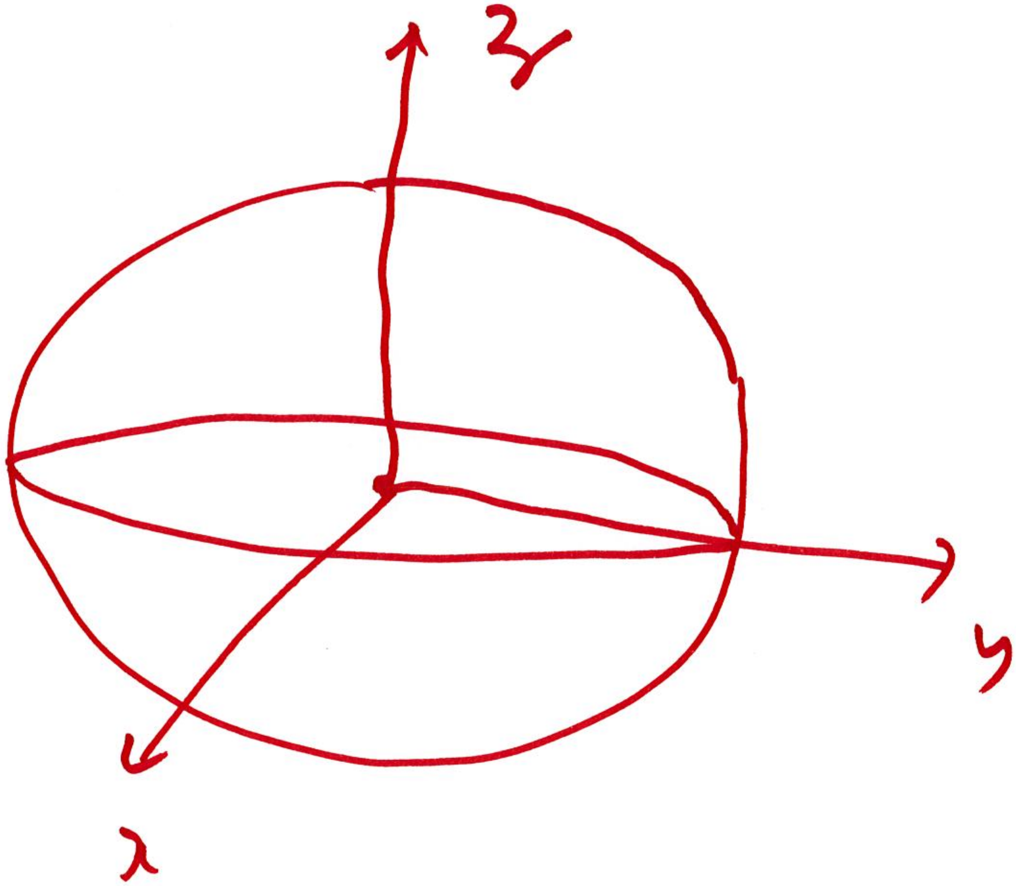
z
y
x

$$x^2 + z^2 = 9$$

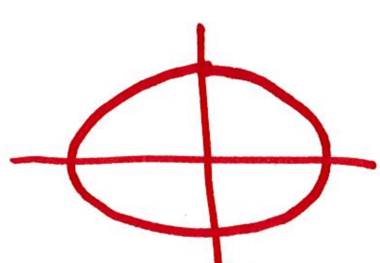


~~Quadrics~~ <u>Quadrics</u>

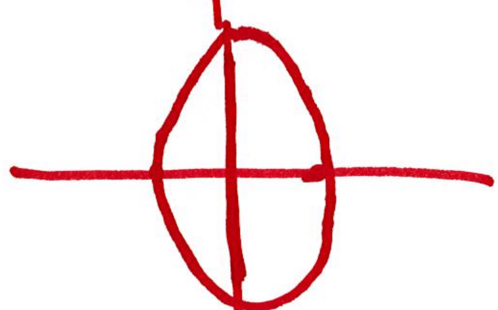
<u>Example:</u> $x^2 + y^2 + z^2 = 1.$



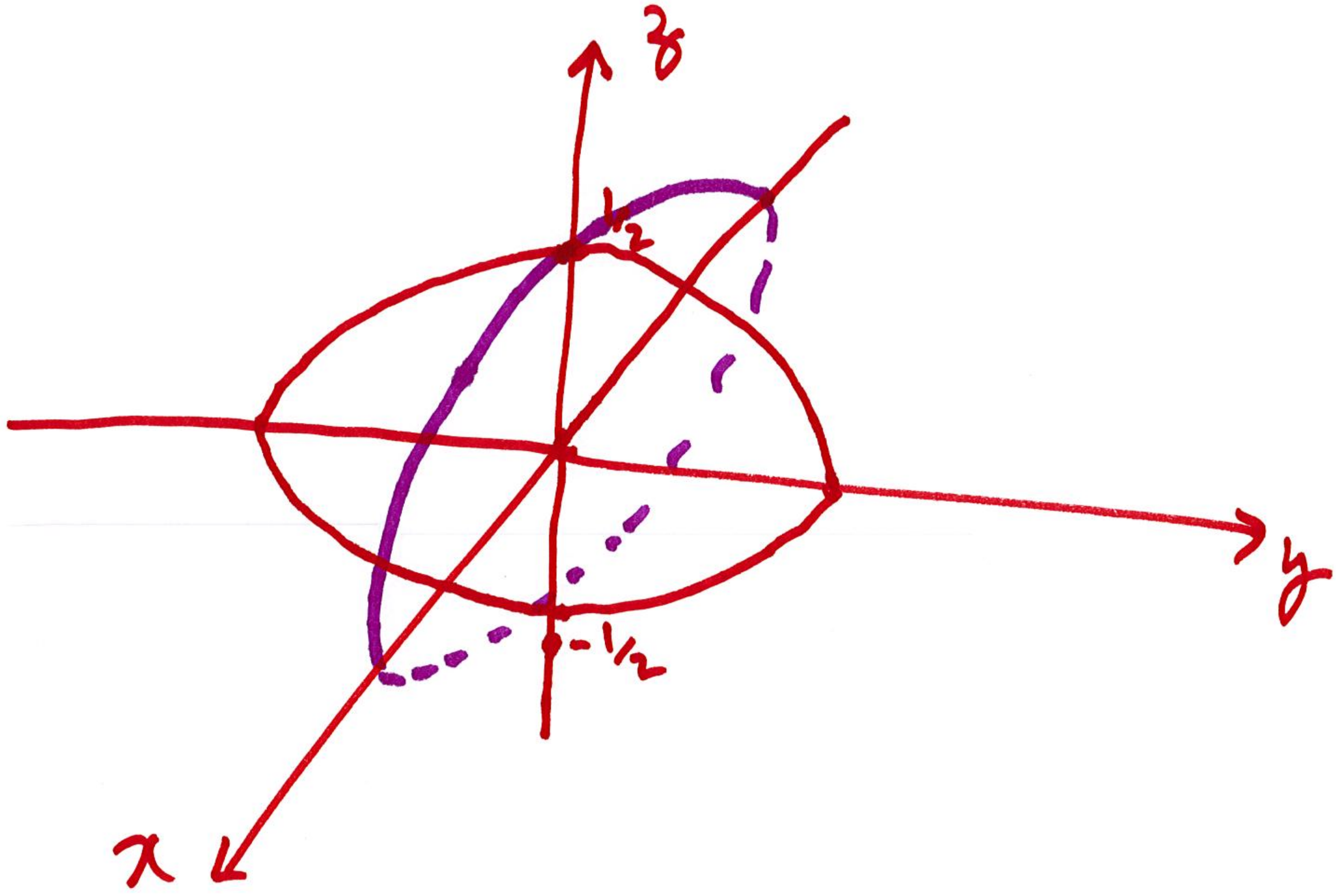
$x^2 + y^2 = 1$



$4x^2 + 2y^2 = 1.$



$$2x^2 + 3y^2 + 4z^2 = 1.$$



$x=0$: $3y^2 + 4z^2 = 1.$ $y=0$, $4z^2=1$

$z^2 = 1/4$

$z = \pm 1/2$

$z=0$, $3y^2=1$

$y^2 = 1/3$

$y = \pm 1/\sqrt{3}$