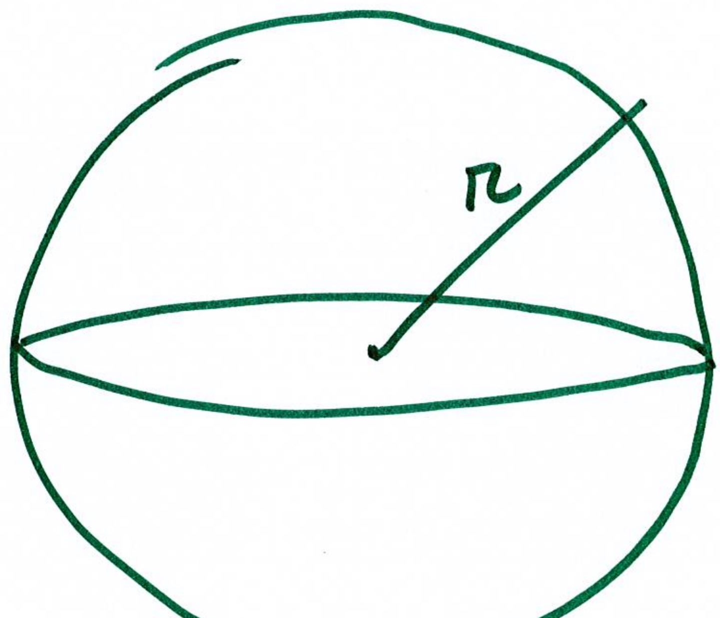
Lesson 32 Section 16.6 Part I

Parametric Surfaces and Areas

Example 1

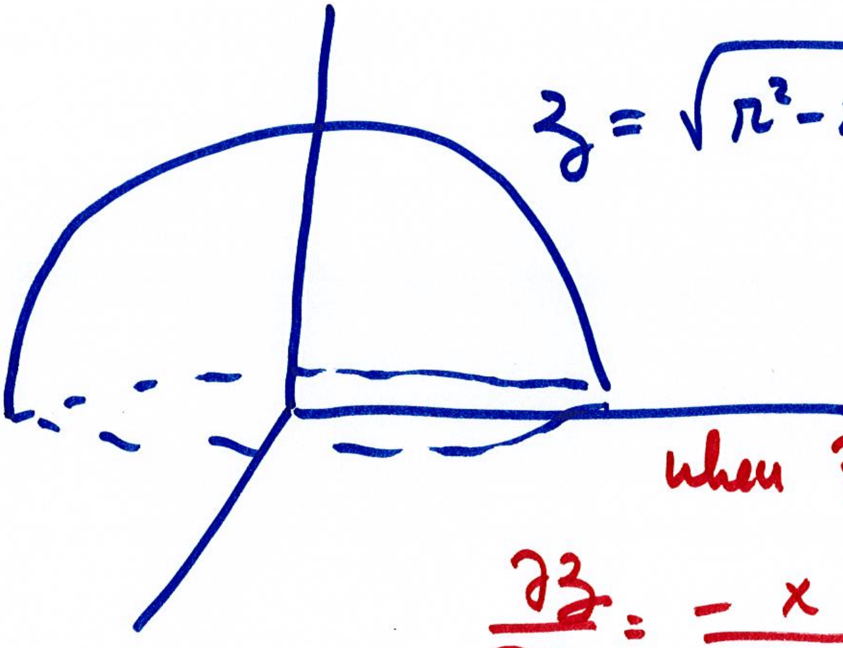


Sphere centered at 0 radius r.

$$x^2+y^2+z^2=r^2.$$

$z=\sqrt{r^2-x^2-y^2}$. When $z>0$.

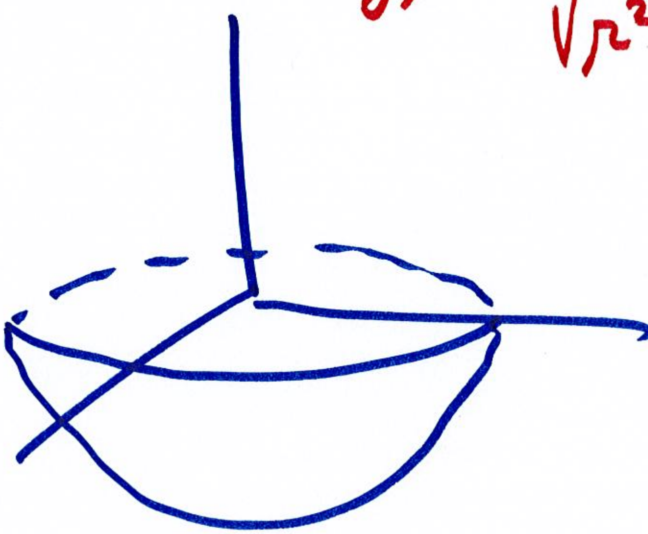
$z = -\sqrt{r^2 - x^2 - y^2}$ when $z < 0$



$z = \sqrt{r^2 - x^2 - y^2}$

when $z = 0$

$$\frac{\partial z}{\partial x} = \frac{-x}{\sqrt{r^2 - \cancel{z}^2 - y^2}}$$



when

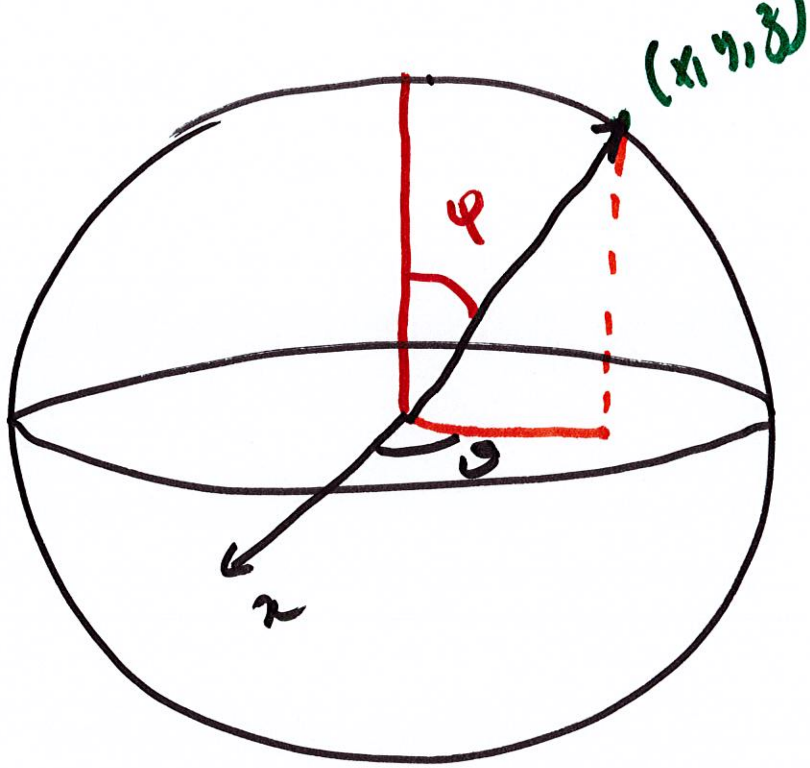
$r^2 - \cancel{z}^2 - y^2 = 0$

$z = -\sqrt{r^2 - x^2 - y^2}$

$$\partial z / \partial x = \frac{x}{\sqrt{r^2 - x^2 - y^2}}$$

not defined when $r^2 - x^2 - y^2 = 0$

Spherical Coordinates



$r =$ radius

$x = r \sin\varphi \cos\theta \qquad 0 \le \varphi \le \pi$

$y = r \sin\varphi \sin\theta \qquad 0 \le \theta < 2\pi$

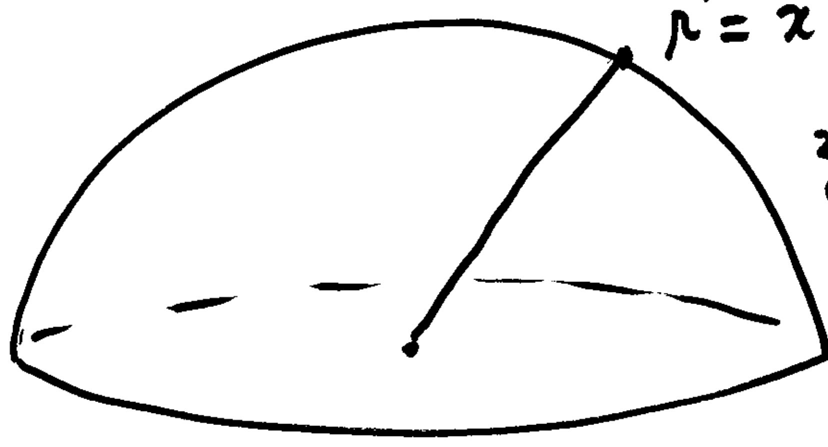
$z = r \cos\varphi$

$$\vec{r}(\theta,\varphi) = r\sin\varphi\cos\theta\,\vec{i} + r\sin\varphi\sin\theta\,\vec{j} + r\cos\varphi\,\vec{k}.$$

Parametric representation of the sphere.

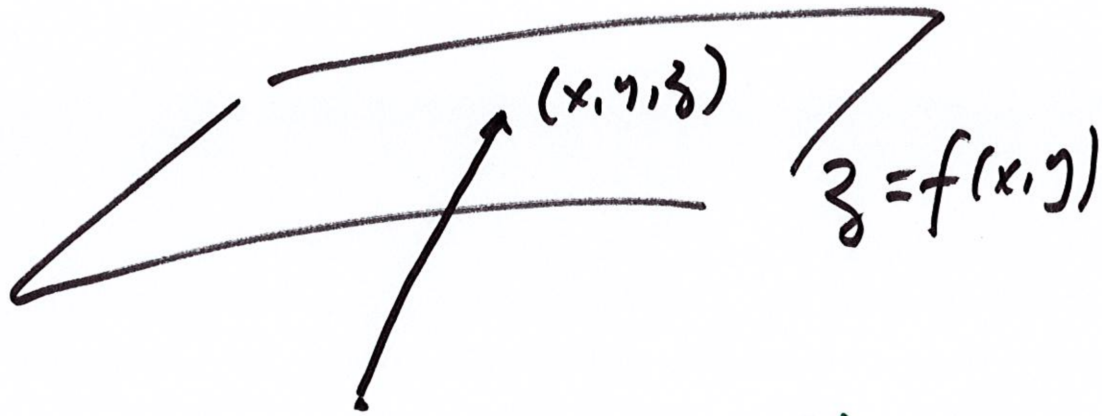
$$z = \sqrt{r^2 - x^2 - y^2}$$

$$\vec{r}(x,y) = x\,\vec{i} + y\,\vec{j} + \sqrt{r^2 - x^2 - y^2}\,\vec{k}$$



Examples: Any graph

$$z = f(x,y)$$

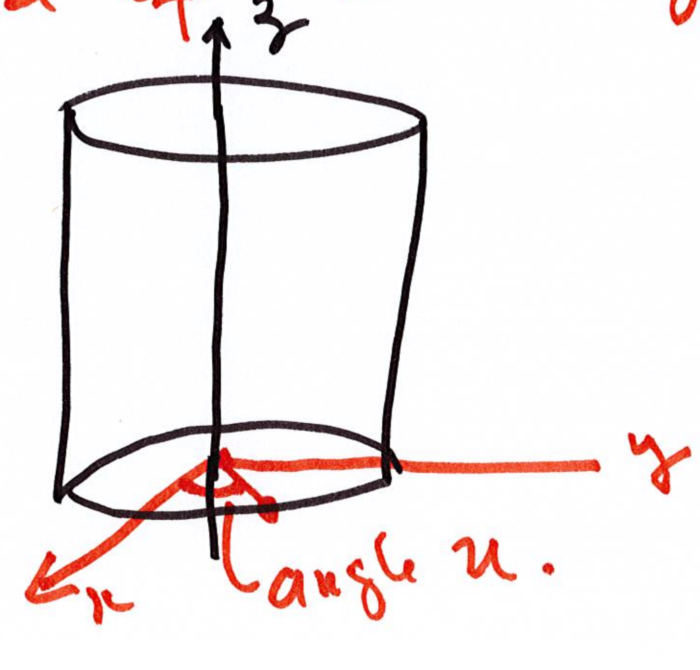


$$\vec{r}(x,y) = x\vec{i} + y\vec{j} + f(x,y)\vec{k}$$

Some times one uses u and v instead of x and y.

Cylinder:

$x^2+y^2 = 16$.



Find a parametric equation for this cylinder.

$$x = 4\cos u \ , \quad y = 4\sin u$$

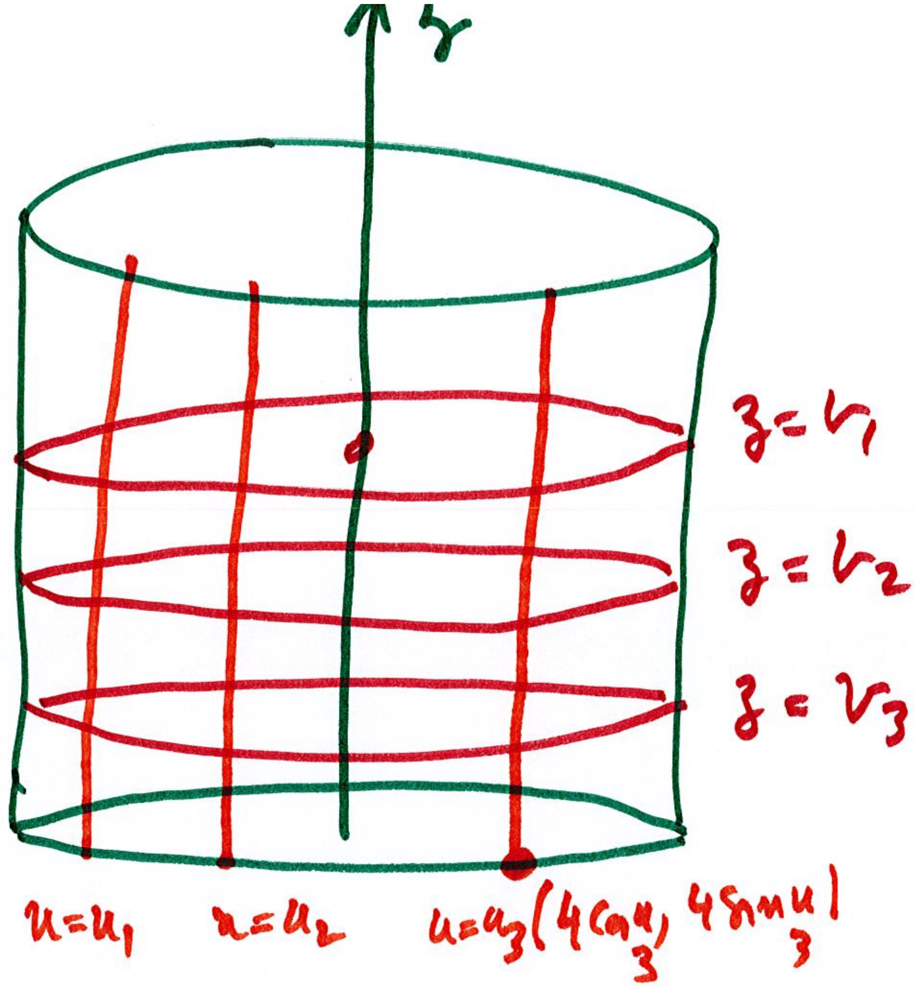
$$z = v$$

$$\vec{r}(u,v) = 4\cos u\,\vec{i} + 4\sin u\,\vec{j} + v\,\vec{k}.$$

Remark: We need two parameters to represent a surface.

Special curves on the surface called "grid curves"

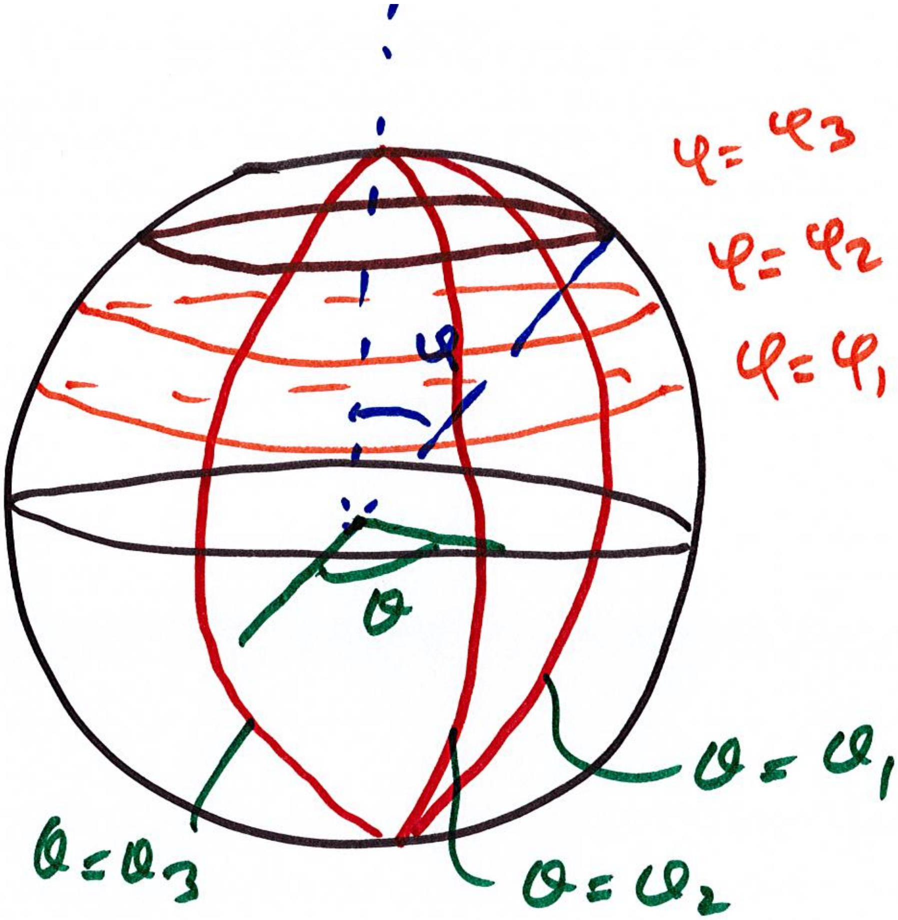
$u =$ constant, $\quad v =$ constant.



<u>Grid curves</u>: v = Constant

$z = v =$ Constant

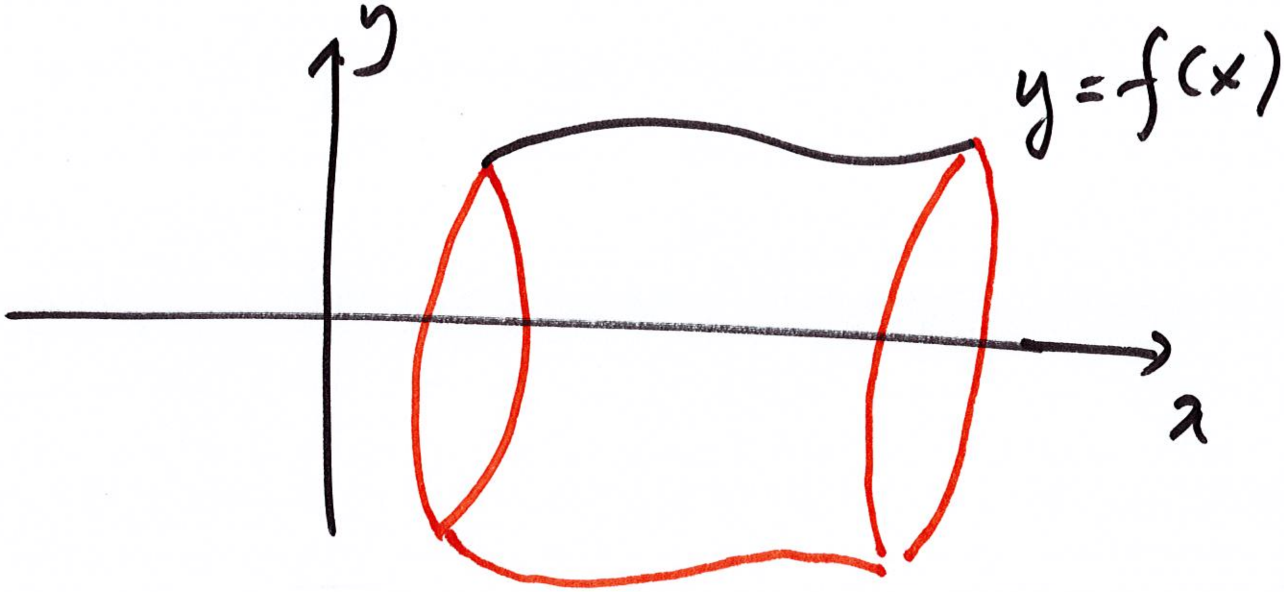
u = Constant $\quad x = 4\cos u,\ y = 4\sin u$



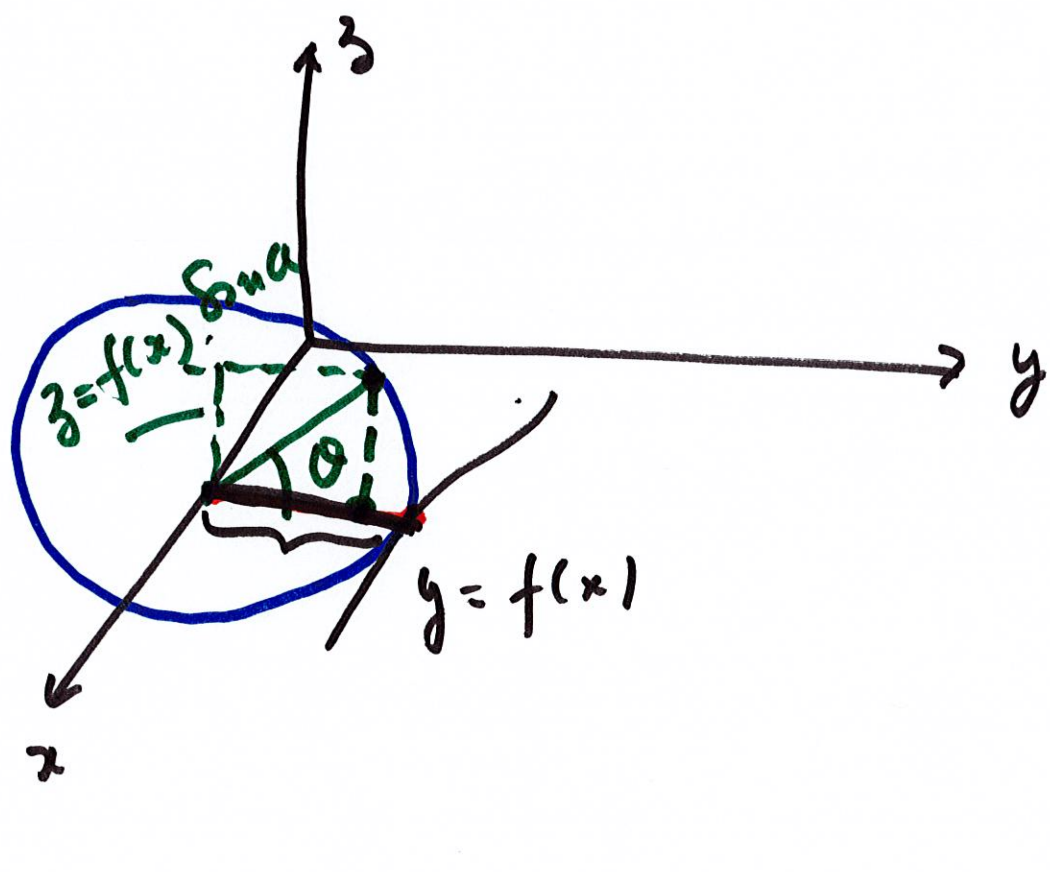
$\varphi =$ constant

Grid curves for the sphere.

Surfaces of Revolution:



Find an equation for such surface.



We need two parameters:

x and θ

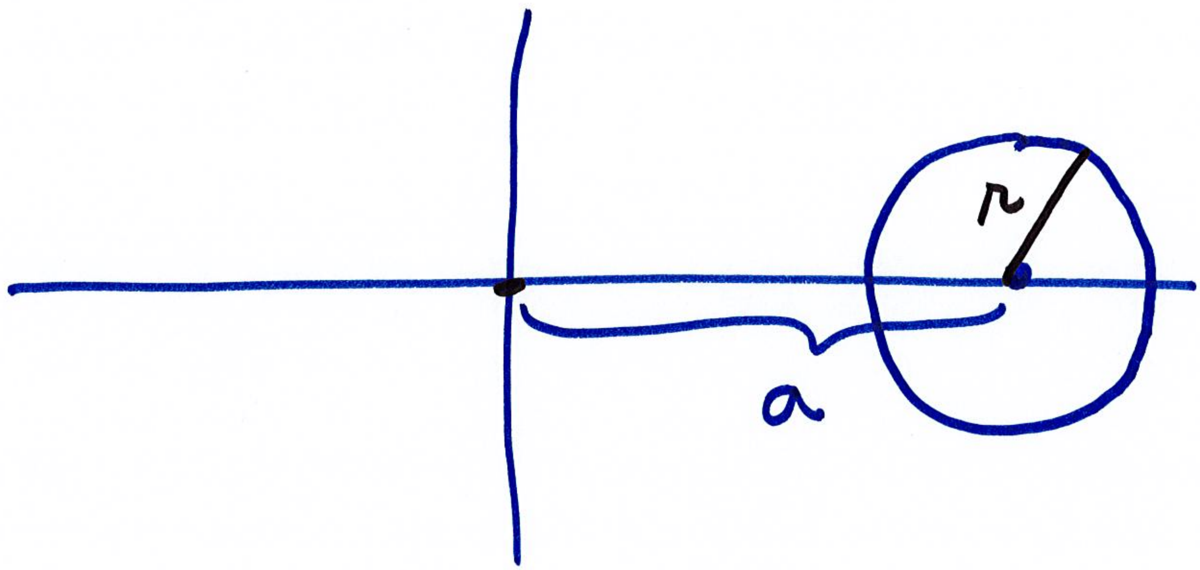
radius of the circle

$= f(x)$

$y = f(x)\cos\theta, \quad z = f(x)\sin\theta.$

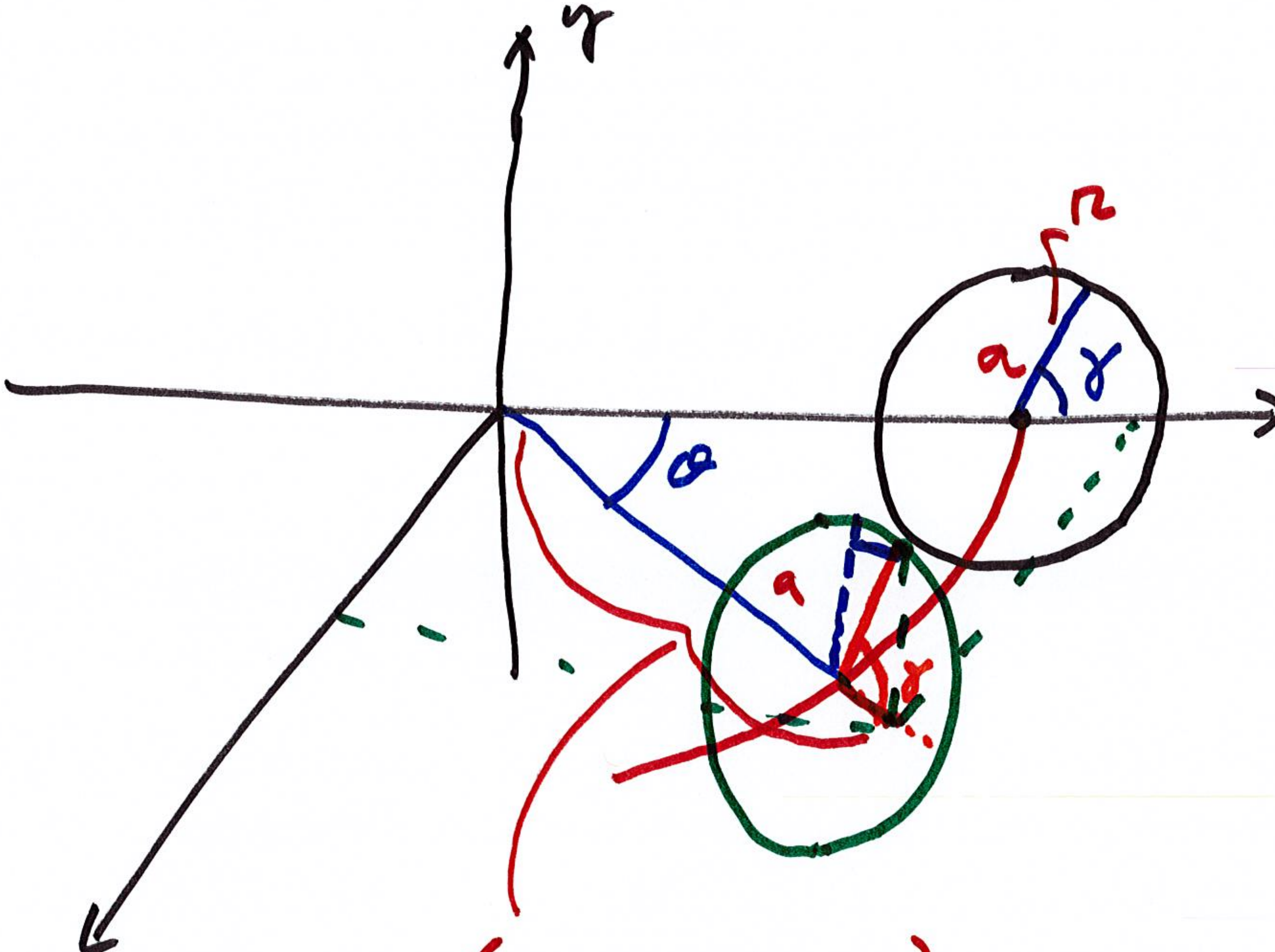
$$\vec{r}(x,\theta) = x\vec{i} + f(x)\cos\theta\,\vec{j} + f(x)\sin\theta\,\vec{k}.$$

Doughnut.



rotate this circle about
the y-axis.

$r < a$



$$(a + r\cos\gamma)\cos\theta = x$$

$$(a + r\cos\gamma)\sin\theta = z$$

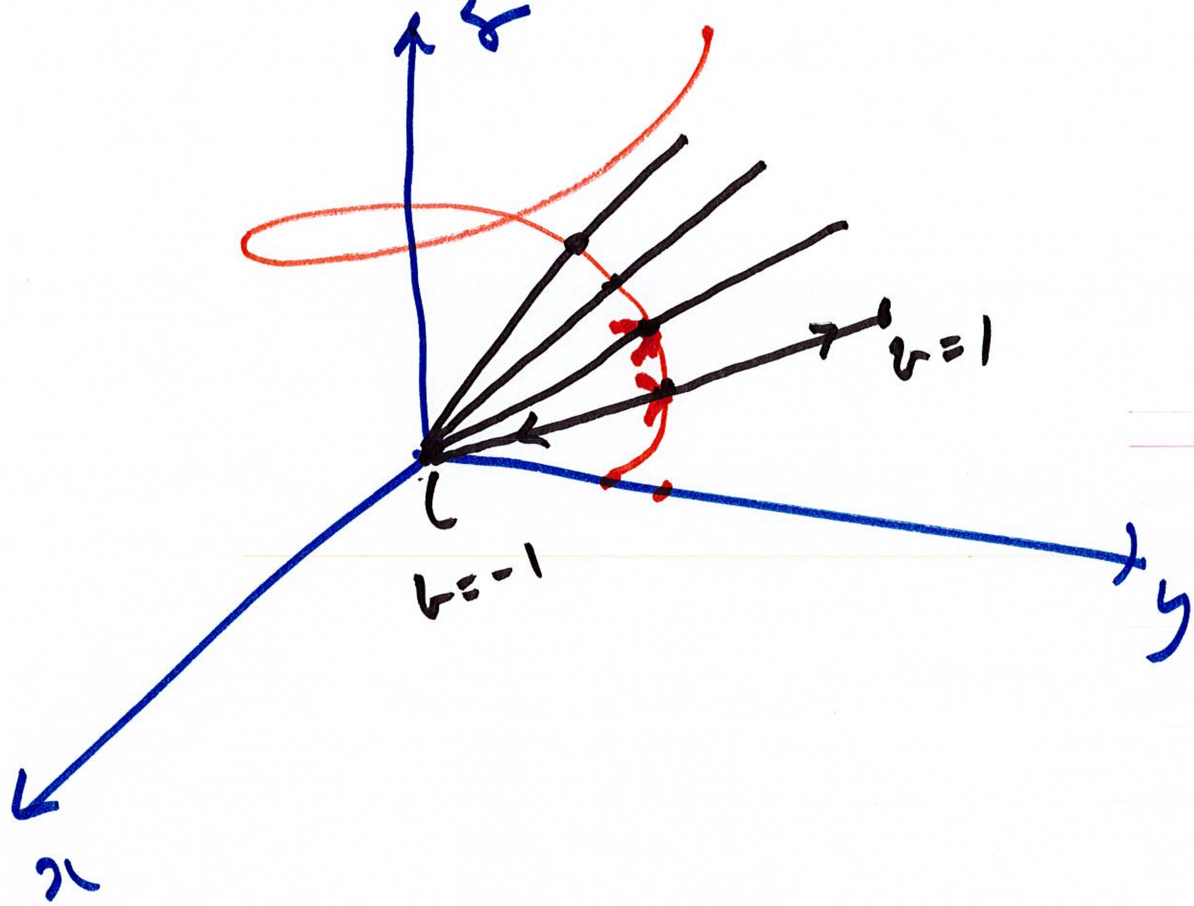
$$y = r\sin\gamma.$$

$$\vec{r}(\gamma,\theta) = (a + r\cos\gamma)\cos\theta\,\vec{i}$$

$$+ (a + r\cos\gamma)\sin\theta\,\vec{j} + r\sin\gamma\,\vec{k}.$$

$$0 \le \theta \le 2\pi\ ,\ 0 \le \gamma \le 2\pi.$$

Ramp



Start with the curve.

$$\vec{r}(u) = \cos u\,\vec{i} + \sin u\,\vec{j} + u\,\vec{k}$$

The way I draw this <u>ramp</u>

at each pt $\vec{r}(u)$ add a multiple of $\vec{r}\,'(u)$.

$$\vec{r}(u,v) = \vec{r}(u) + v\,\vec{r}'(u)$$

when $v=-1$ $\quad \vec{r}(u,-1)=0$

$v=1$ $\quad \vec{r}(u,v) = 2\vec{r}(u)$

$$\vec{r}(u,v) = (1+v)\,\vec{r}(u)$$

$$-1 \le v \le 1$$

and $\quad 0 \le u \le 10$