Lesson 39 Review for the final Exam.

#18 Practice Problems

$$f(x,y) = x^2 y$$

Find $\vec{u}$, $D_{\vec{u}} f(2,3) = 0$

Solution: $\vec{u} = u_1 \vec{i} + u_2 \vec{j}$.

$$D_{\vec{u}} f = \vec{u} \cdot \vec{\nabla} f.$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x} \vec{i} + \frac{\partial f}{\partial y} \vec{j}.$$

$$\vec{\nabla} f = 2xy \vec{i} + x^2 \vec{j}$$

$$\vec{\nabla} f(2,3) = 12 \vec{i} + 4 \vec{j}.$$

$$\vec{u} \cdot \vec{\nabla} f = 12 u_1 + 4 u_2 = 0.$$

$$4 u_2 = -12 u_1 \quad u_2 = -3 u_1.$$

$$\vec{u} = u_1 \vec{i} - 3u_1 \vec{j}$$

$$\vec{u} = u_1 (\vec{i} - 3\vec{j}).$$

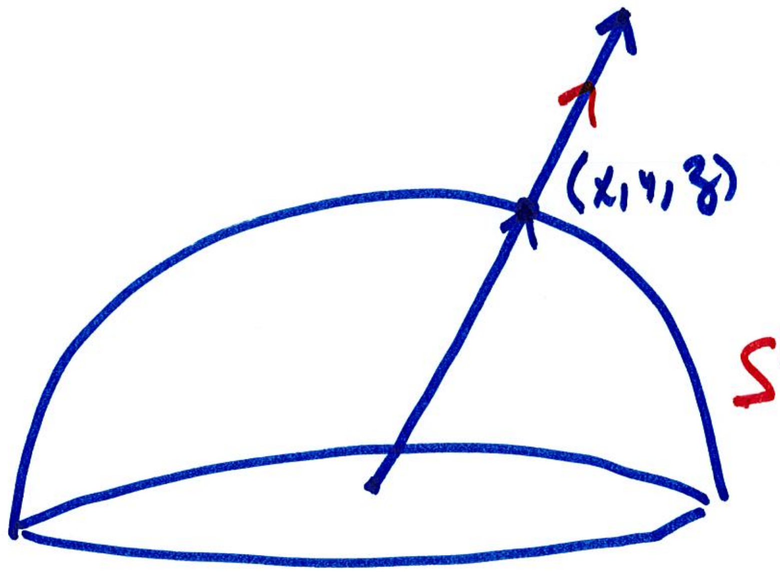
$$|\vec{u}| = |u_1| \sqrt{1 + 9}$$

$$= \sqrt{10}\, |u_1| = 1$$

$$|u_1| = 1/\sqrt{10}$$

$$\boxed{\vec{u} = \frac{1}{\sqrt{10}} (\vec{i} - 3\vec{j})} \quad a$$

$$\vec{u} = -\frac{1}{\sqrt{10}} (\vec{i} - 3\vec{j}).$$



$\vec{x} = x\vec{i} + y\vec{j} + z\vec{k}$ is normal to S

$$\vec{n} = -\frac{\vec{x}}{|\vec{x}|}$$

$$\vec{F} \cdot \vec{n} = z\frac{\vec{x}}{|\vec{x}|} \cdot -\frac{\vec{x}}{|\vec{x}|}$$

$$= -z\,\frac{\vec{x} \cdot \vec{x}}{|\vec{x}|^2} = -z\,\frac{\cancel{|\vec{x}|^2}}{\cancel{|\vec{x}|^2}}$$

$$= -z.$$

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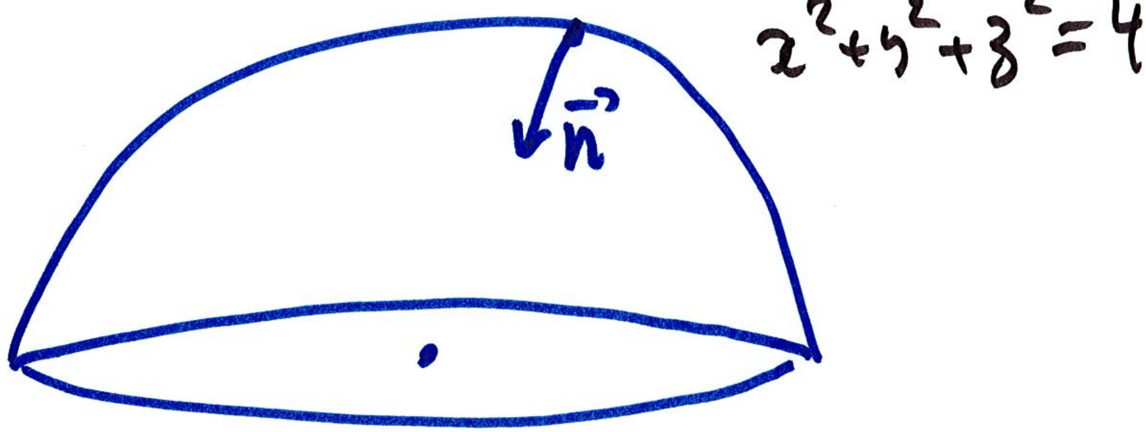


$$\vec{F} = z\frac{\cancel{\vec{x}}}{|\cancel{\vec{x}}|} \qquad \vec{F} = z\frac{\vec{x}}{|\vec{x}|}$$

$$\vec{x} = x\vec{i} + y\vec{j} + z\vec{k}.$$

$$\boxed{\iint_S \underbrace{\vec{F}\cdot\vec{n}}\, dS = \iint_S -z\, dS.}$$

$$\text{Area} = R\,d\varphi \cdot R\sin\varphi\,d\alpha$$

$$\boxed{dS = R^2 \sin\varphi\,d\varphi\,d\alpha.}$$

$\underline{R=2}$ $\quad\quad$ $z = R\cos\varphi$

$$\iint_S z\,dS = \iint 2\cos\varphi \cdot 2^2 \sin\varphi\,d\varphi\,d\alpha$$

$$= 8\int_0^{2\pi}\int_0^{\pi/2} \sin\varphi \cos\varphi\,d\varphi\,d\alpha$$

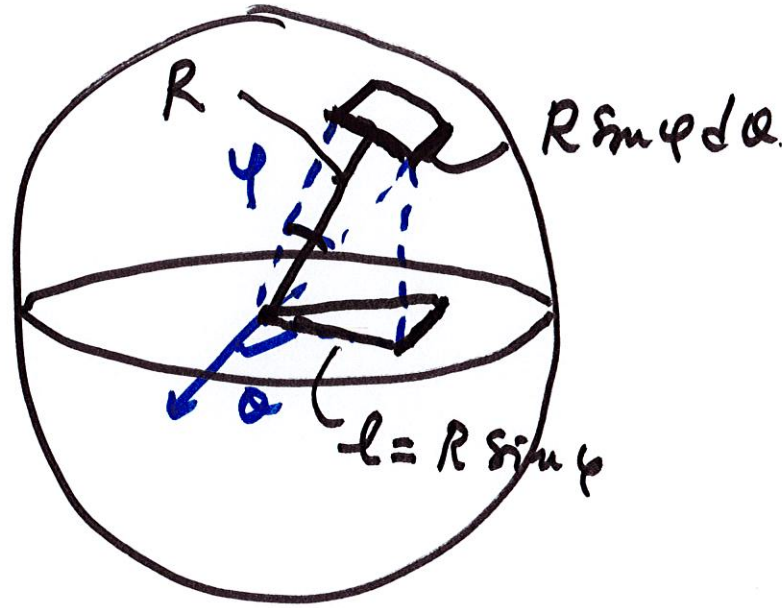
$$= 16\pi \int_0^{\pi/2} \sin\varphi\cos\varphi\,d\varphi.$$

$\sin\varphi = u,\ du = \cos\varphi\,d\varphi$

$$= 16\pi \int_0^1 u\,du = \underline{8\pi}$$

$$\iint_S z\, dS =$$

$dS =$ sphere



Parametrize the sphere

$x = 2 \sin\varphi \cos\theta$

$y = 2 \sin\varphi \sin\theta$

$z = 2 \cos\varphi$

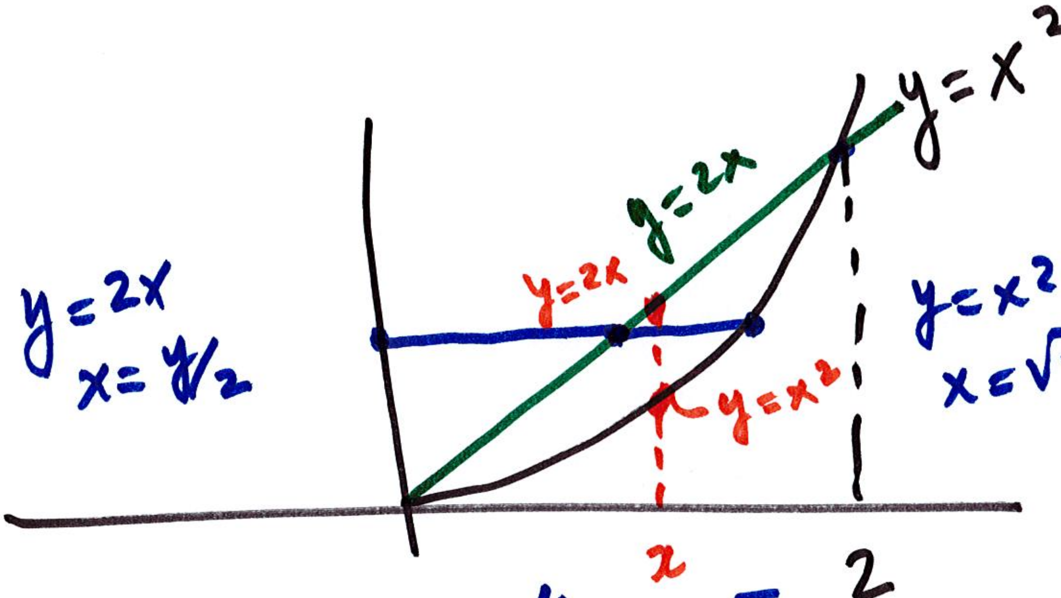
$\vec{r}(\varphi, \theta) = x\vec{i} + y\vec{j} + z\vec{k}$

$$\left| \frac{\partial \vec{r}}{\partial \varphi} \times \frac{\partial r}{\partial \theta} \right|$$

28) $$\int_0^4 \int_a^b f(x,y)\,dx\,dy =$$

$$= \int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx$$

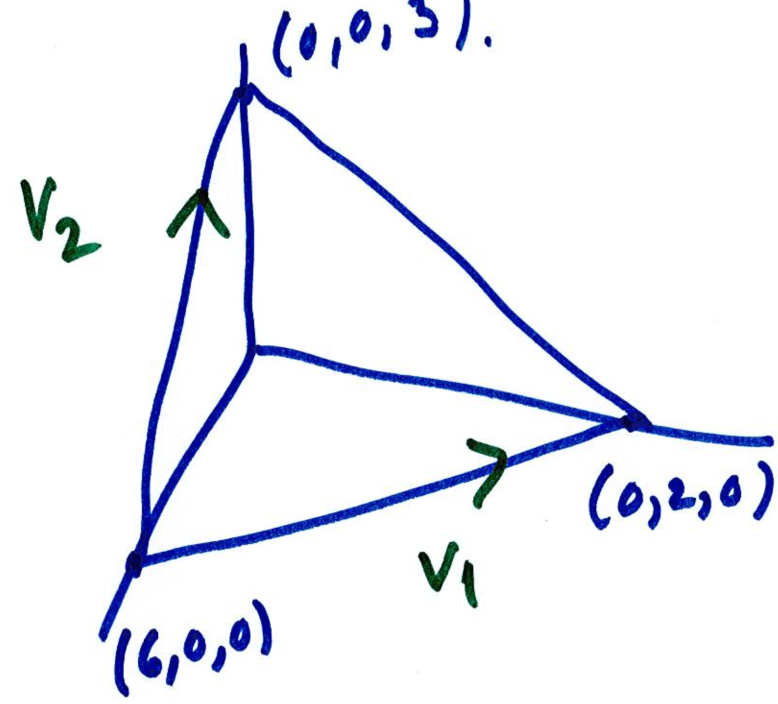
Find $\underline{\underline{a}}$ and $\underline{\underline{b}}$



$$\int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx = \int_0^4 \int_{y/2}^{\sqrt{y}} f(x,y)\,dx\,dy$$

#32) $x + 3y + 2z = 6$

Find the area of the portion of this plane that lies in the 1st octant



Solution 1: (No Calculus)

$V_1 = -6\vec{i} + 2\vec{j}$, $V_2 = -6\vec{i} + 3\vec{k}$

$$\text{Area} = \frac{1}{2} |V_1 \times V_2|.$$

This would not work if we wanted to Find $\iint_S x \, dS$.

Solution 2: (Use Calculus)

Surface: $x = u, \; y = v$

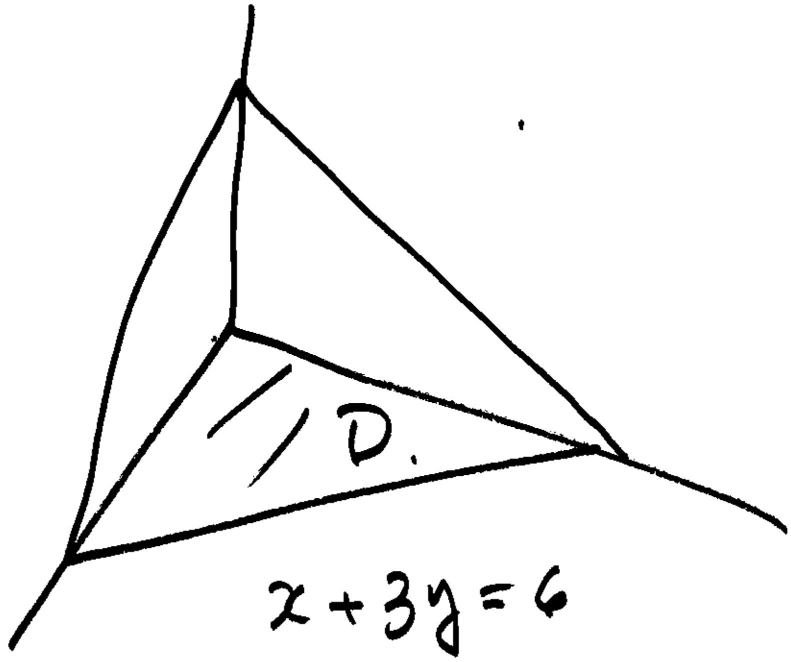
$$z = 3 - \frac{x}{2} - \frac{3y}{2}$$

$$z = 3 - \frac{u}{2} - \frac{3v}{2}$$

$$\vec{r}(u,v) = u\,\vec{i} + v\,\vec{j} + \left(3 - \frac{u}{2} - \frac{3v}{2}\right)\vec{k}.$$

$$\frac{\partial \vec{r}}{\partial u} = \vec{i} - \frac{1}{2}\vec{k} \; ; \; \frac{\partial \vec{r}}{\partial v} = \vec{j} - \frac{3}{2}\vec{k}.$$

$$\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1/2 \\ 0 & 1 & -3/2 \end{vmatrix}.$$



$$\iint_D \left| \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right| dA$$