Lesson 4 Section 12.6

(Continuation)

Quadrics

Example: Spheres

$$4x^2 + 4y^2 + 4z^2 - 3x + 2y + 8z = 10$$

Step 1: Divide by 4

$$x^2 + y^2 + z^2 - \frac{3}{4}x + \frac{1}{2}y + 2z = \frac{10}{4}$$

$$x^2 - \tfrac{3}{4}x = \left(x - \tfrac{3}{8}\right)^2 - \frac{9}{64}$$

$$y^2 + \tfrac{1}{2}y = \left(y + \tfrac{1}{4}\right)^2 - \tfrac{1}{16}$$

$$z^2 + 2z = (z+1)^2 - 1.$$

$$\left(x-\tfrac{3}{8}\right)^2 - \tfrac{9}{64} + \left(y+\tfrac{1}{4}\right)^2 - \tfrac{1}{16} + (z+1)^2 - 1 = \frac{10}{4}$$

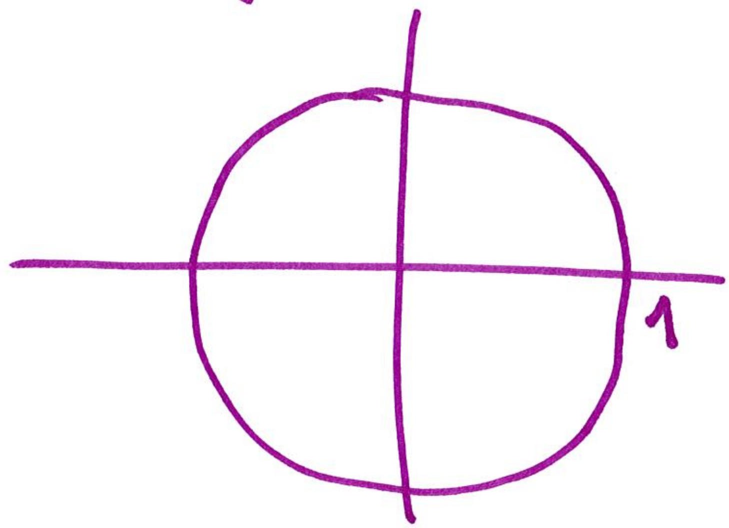
$$\boxed{\left(x-\tfrac{3}{8}\right)^2 + \left(y+\tfrac{1}{4}\right)^2 + (z+1)^2 = \tfrac{10}{4} + 1 + \tfrac{9}{64} + \tfrac{1}{16}}$$

Sphere Centered at $\left(\tfrac{3}{8}, -\tfrac{1}{4}, -1\right)$

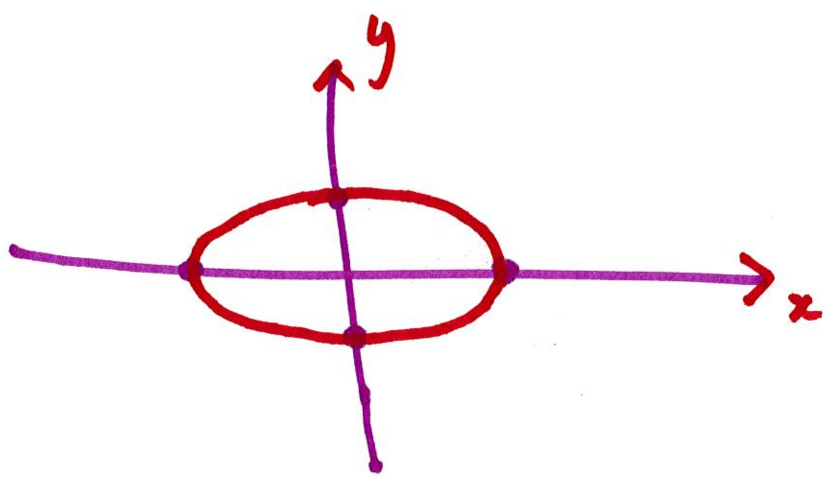
Radius $\sqrt{\tfrac{10}{4} + 1 + \tfrac{9}{64} + \tfrac{1}{16}}$.

2 Dimensions:

Circles: $x^2 + y^2 = 1$



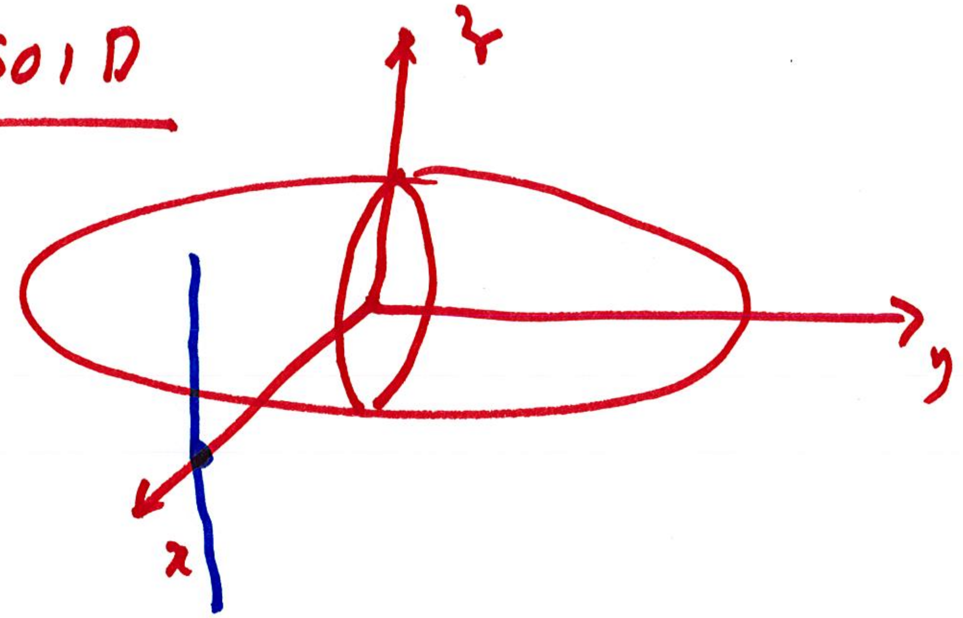
Ellipses: $3x^2 + 4y^2 = 1$



$x = 0, \quad 4y^2 = 1, \quad y^2 = 1/4, \quad y = \pm 1/2$

$y = 0 \quad 3x^2 = 1 \quad x = \pm 1/\sqrt{3}$

ELLIPSOID

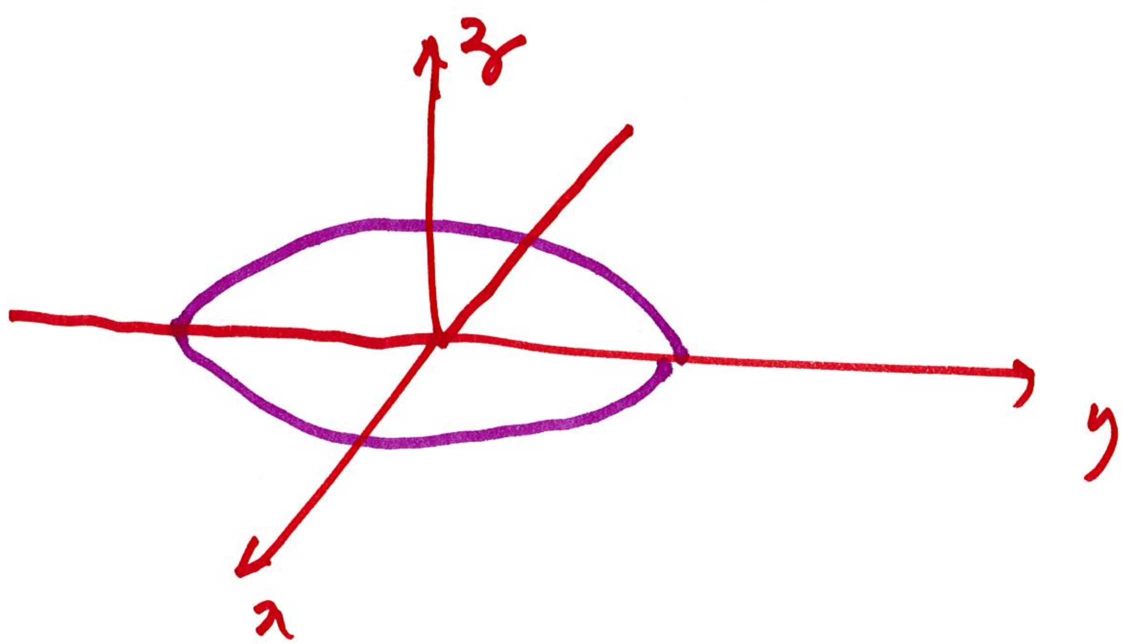


$$A\underline{\underline{x}}^2 + By^2 + Cz^2 = 1.$$

$A > 0$
$B > 0$
$C > 0.$

CROSS SECTIONS:

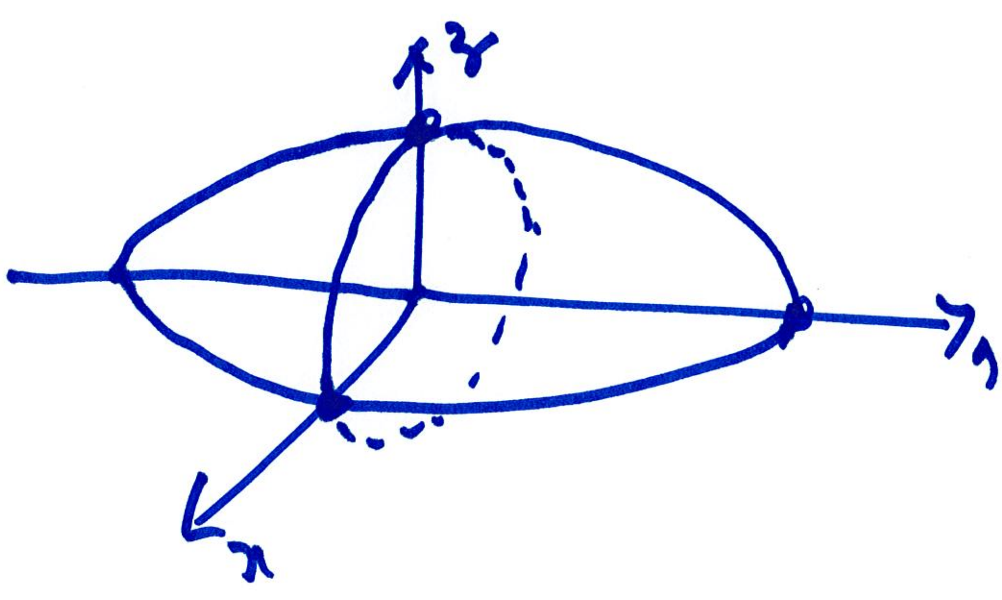
Take $x = 0$: $\quad By^2 + Cz^2 = 1. \leftarrow$



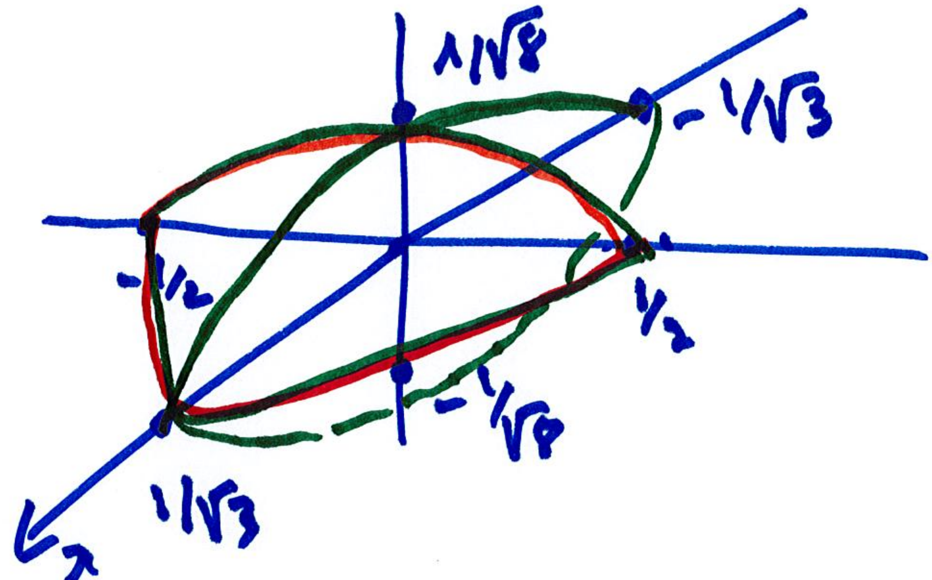
$$\underbrace{By^2 + Cz^2} = \underbrace{1 - Ax^2}$$

If $1 - Ax^2 < 0$ No intersection

$B > 0, \quad C > 0$



$$3x^2 + 4y^2 + 8z^2 = 1$$



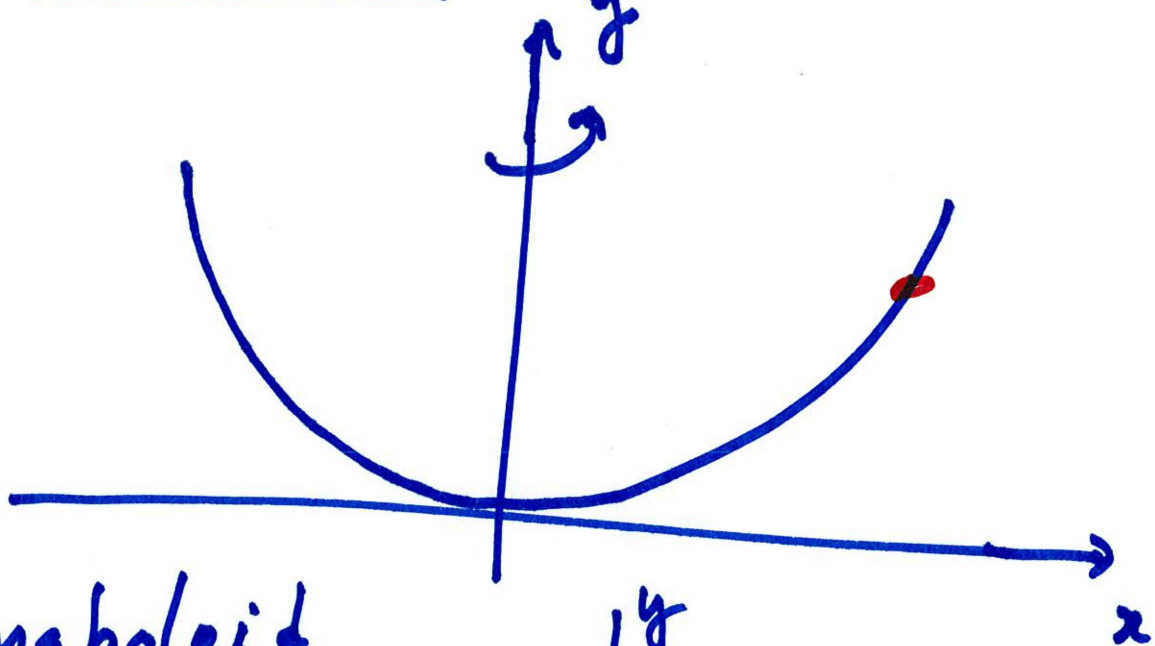
Set $x = y = 0$ $\quad 8z^2 = 1$, $z^2 = 1/8$

$z = \pm 1/\sqrt{8}$

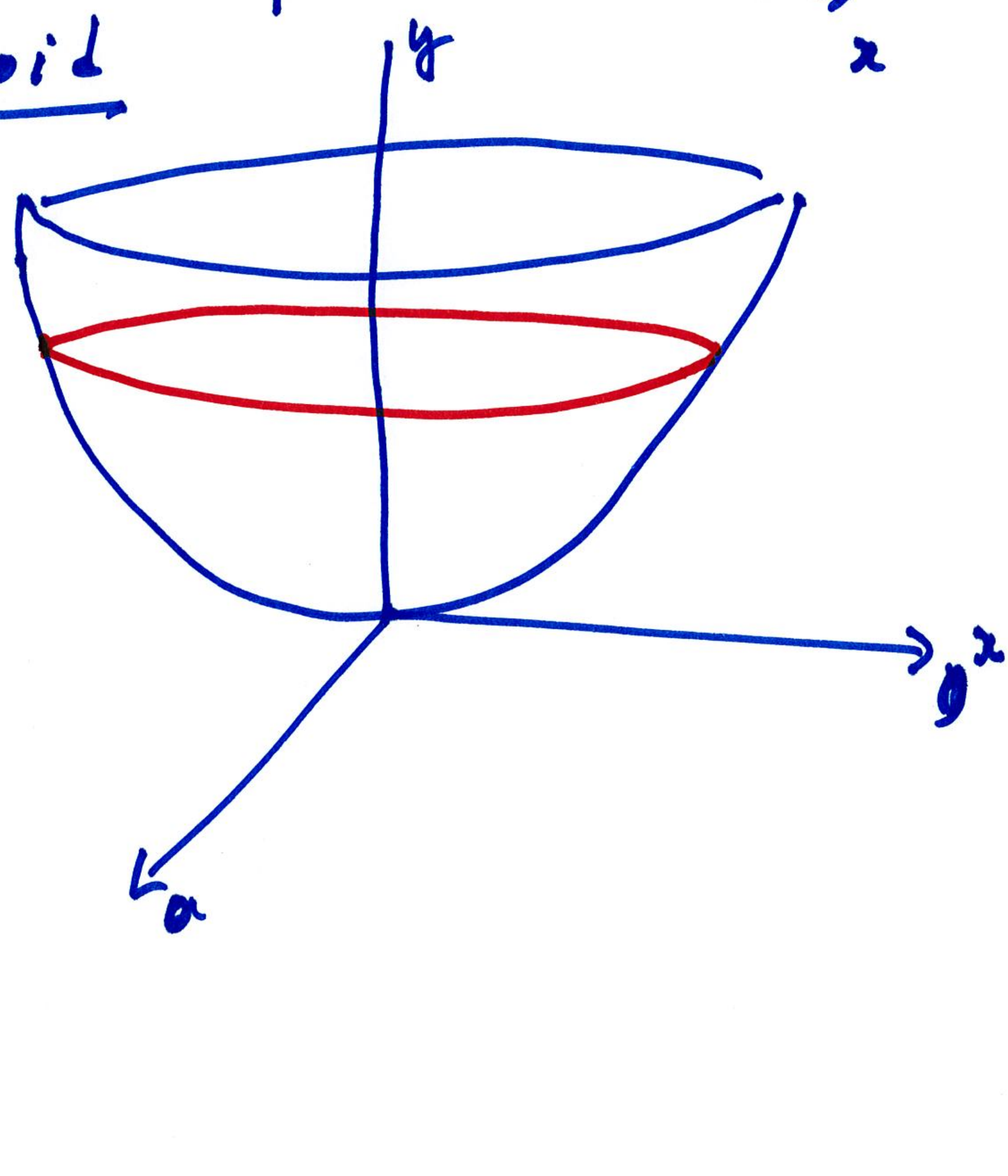
Set $y = z = 0$ $\quad 3x^2 = 1$, $x = \pm \frac{1}{\sqrt{3}}$

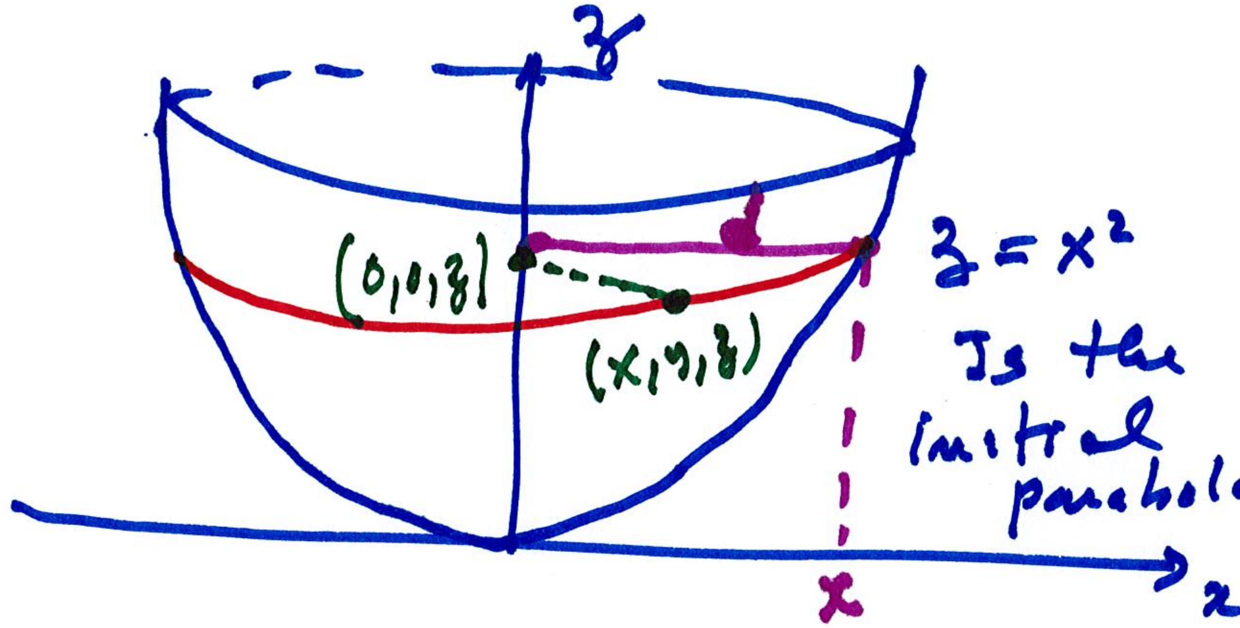
$x = z = 0$ $\quad 4z^2 = 1$ $\quad y = \pm \frac{1}{2}$

Parabolas



Paraboloid





Find an equation for this surface.

Distance between $(0,0,z)$ and (x,y,z)

$$d = \sqrt{(x-0)^2+(y-0)^2+(z-z)^2} = \sqrt{x^2+y^2}.$$

$d^2 = x^2+y^2$. = Radius of the circle.

So on the plane

xz , $d = x$

So $d^2 = x^2 = z$.

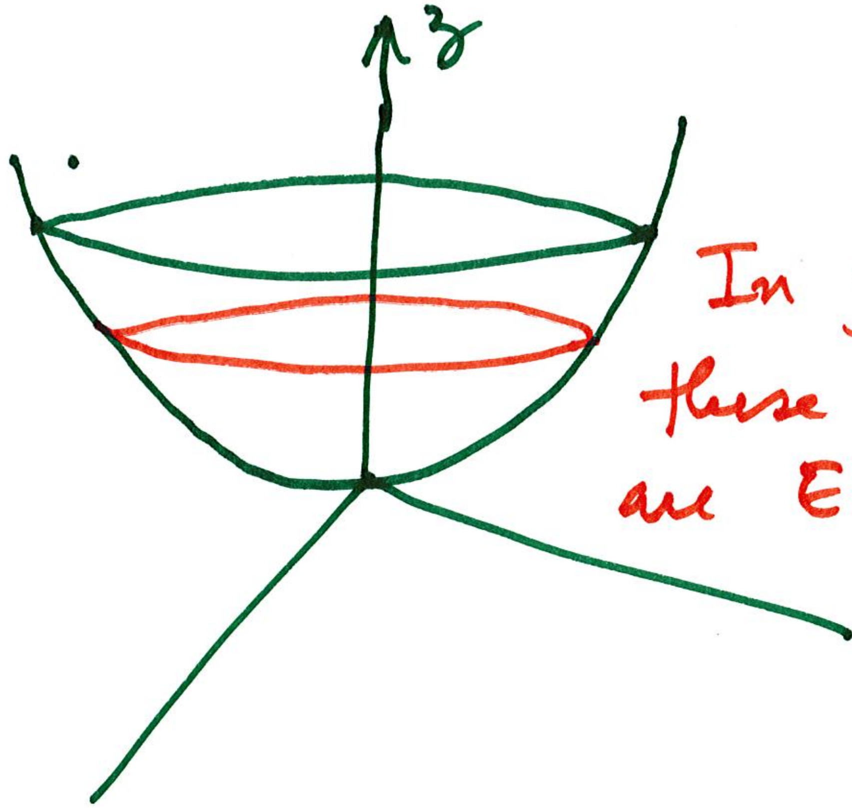
$$\boxed{\text{Equation:} \quad z = x^2 + y^2}$$

Thus is a paraboloid

of <u>Revolution</u>.

In general $z = Ax^2 + By^2$

$A > 0, \quad B > 0$

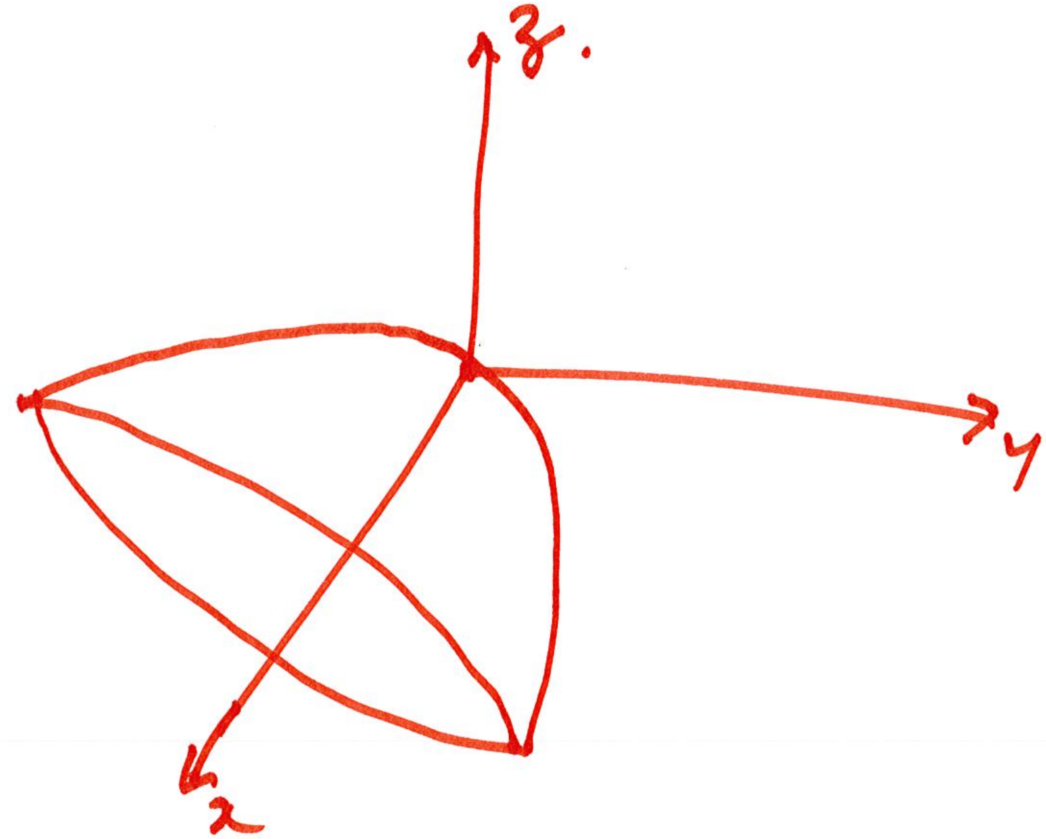


In general these sections are Ellipses

$$z = Ax^2 + By^2 \quad , \quad A, B > 0$$

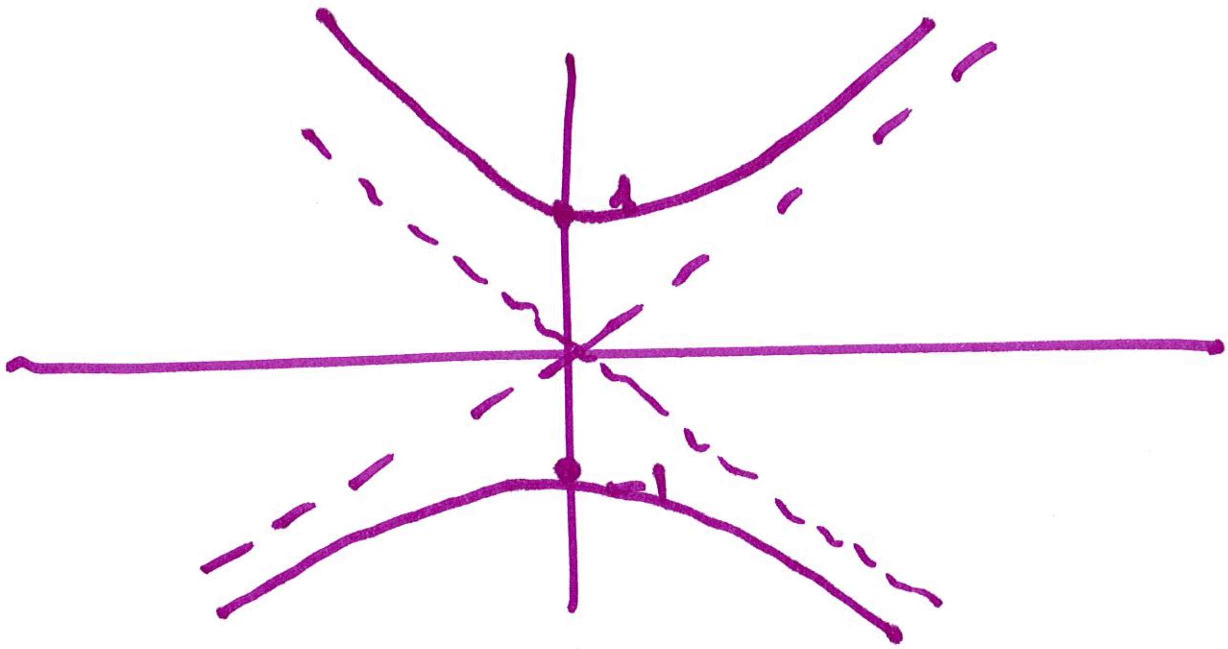
z-axis is the axis of symmetry.

$$x = 2y^2 + 4z^2$$

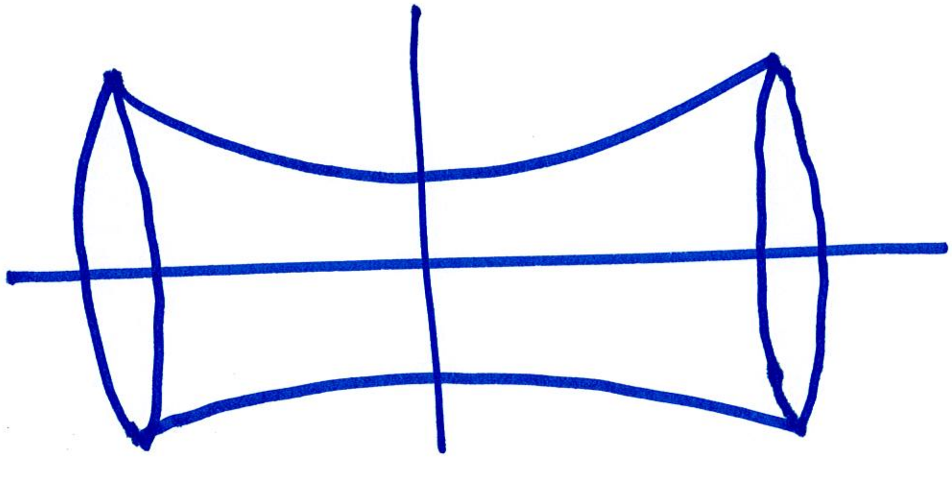


Hyperbolas.

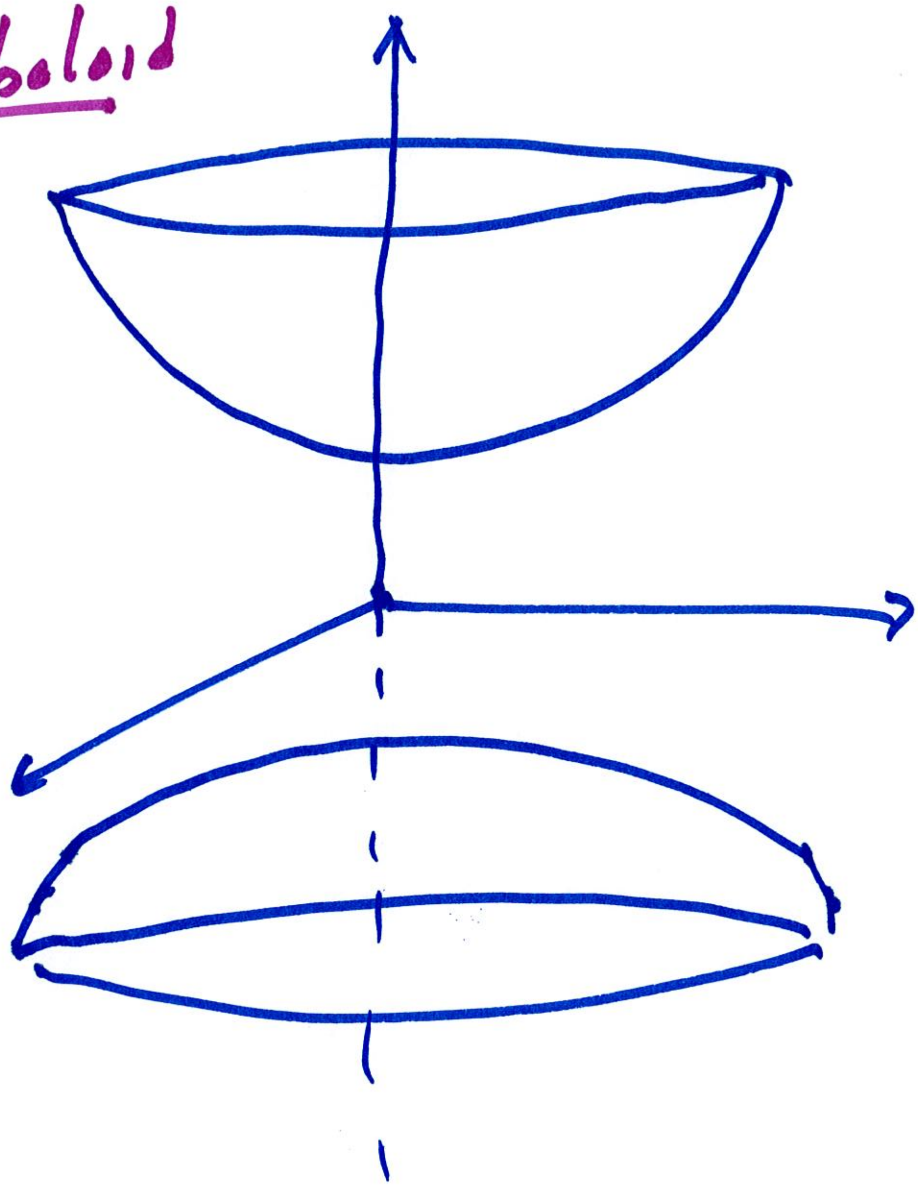
$$y^2 - x^2 = 1.$$



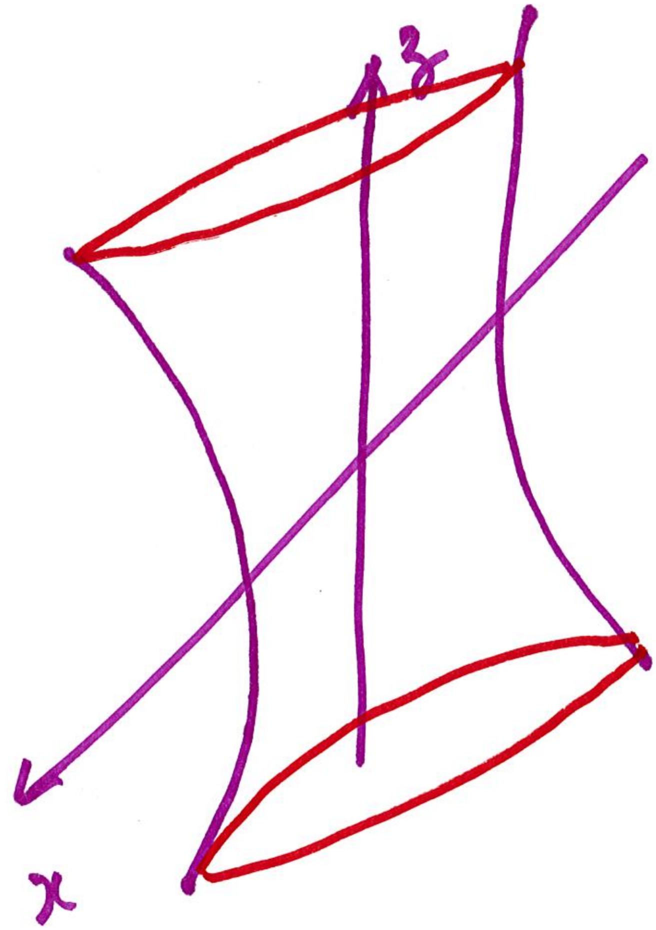
Hyperboloid of one sheet.



Hyperboloid of two sheets



Find Equations for these Surfaces.



$x^2 - z^2 = 1$ Hyperbola.

$x^2 + y^2 - z^2 = 1$ Hyperboloid of one sheet

The negative sign gives the axis of symmetry.

$$Ax^2 + By^2 - Cz^2 = 1$$

$A > 0, \; B > 0, \; C > 0$

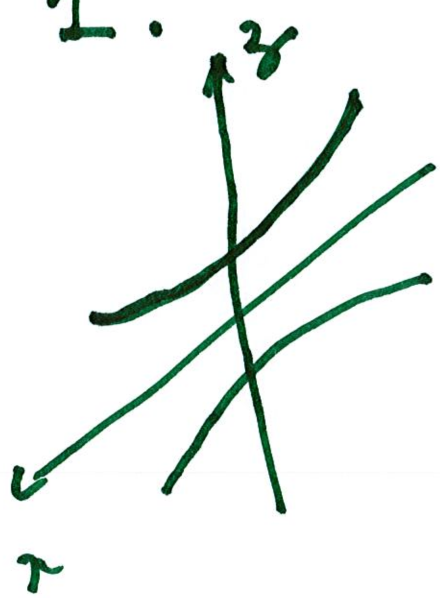
gives a hyperboloid of

<u>one sheet</u>

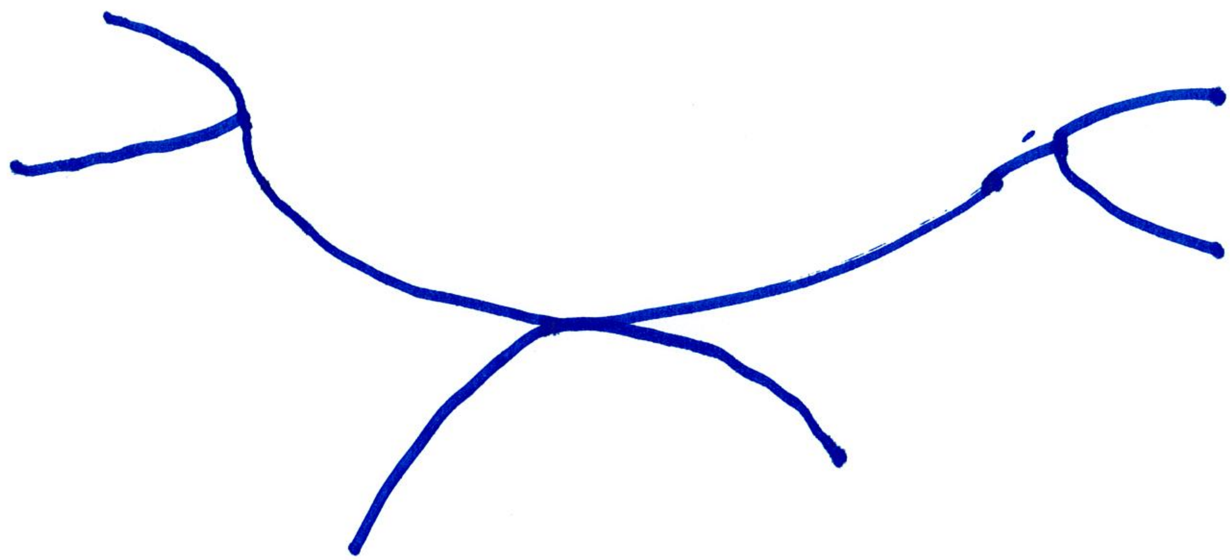
<u>Hyperboloid of two sheets</u>

Axis of symmetry.

$$z^2 - x^2 - y^2 = 1.$$



Hyperbolic Paraboloid



$$z = y^2 - x^2$$

$$z = Ax^2 + By^2, \quad A, B > 0$$

are paraboloids.

$$z = Ax^2 + By^2, \quad A > 0, \; B < 0$$

are hyperbolic Paraboloids.

In general:

$Ax^2 + By^2 + Cz^2 = 1, \quad A, B, C > 0$

are Ellipsoids

$Ax^2 + By^2 + Cz^2 = 1, \quad A, B > 0, \quad C < 0$

are hyperboloids of one sheet

Axis of Symmetry: Variable with negative coefficient

$Ax^2 + By^2 + Cz^2 = 1, \quad A > 0, \; B < 0, \; C < 0$

are hyperboloids of two sheets

Axis of Symmetry: Variable with positive coefficient.