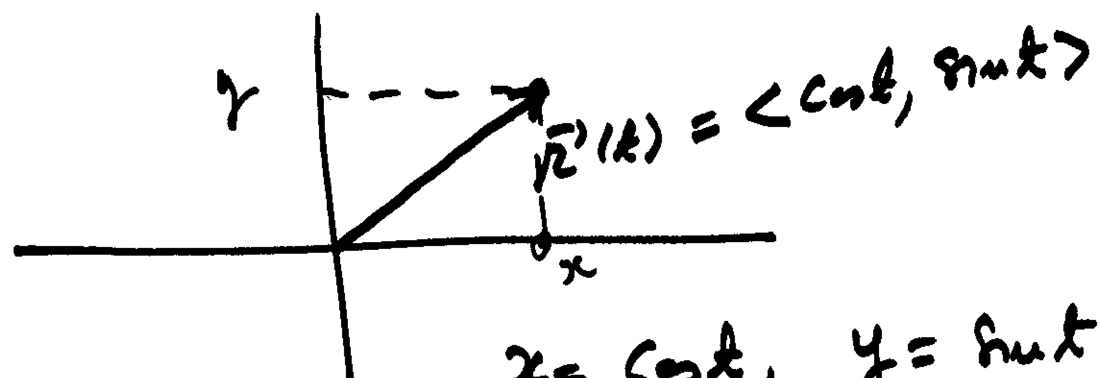
# Lesson 5 Section 13.1

Vector Functions and Space Curves.

## 2 - Dimensions

$\vec{r}(t) = \langle \cos t, \sin t \rangle$

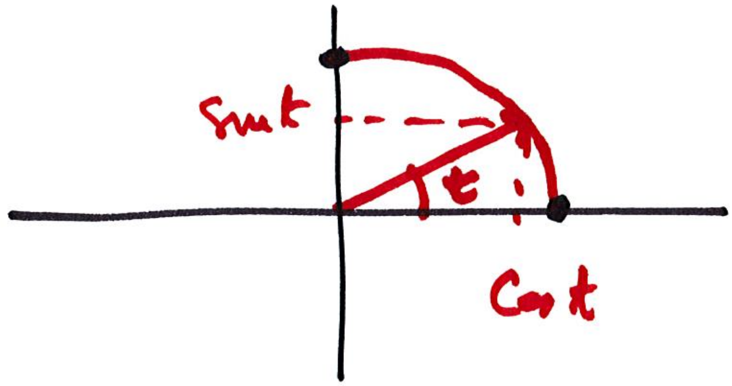



$x = \cos t, \quad y = \sin t.$

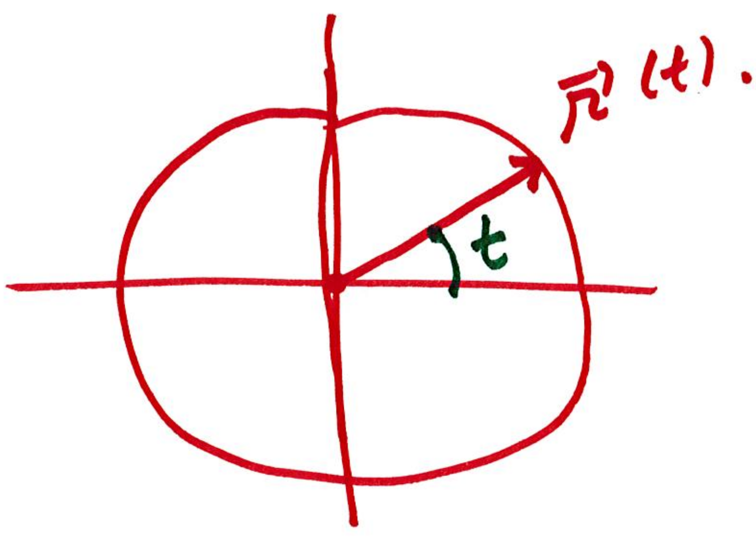
As $\underline{\underline{t}}$ varies, the tip of the vector describes a curve on the plane.

$t = 0, \quad \vec{r}(0) = \langle 1, 0 \rangle$

$t = \pi/2, \quad \vec{r}(\pi/2) = \langle 0, 1 \rangle$



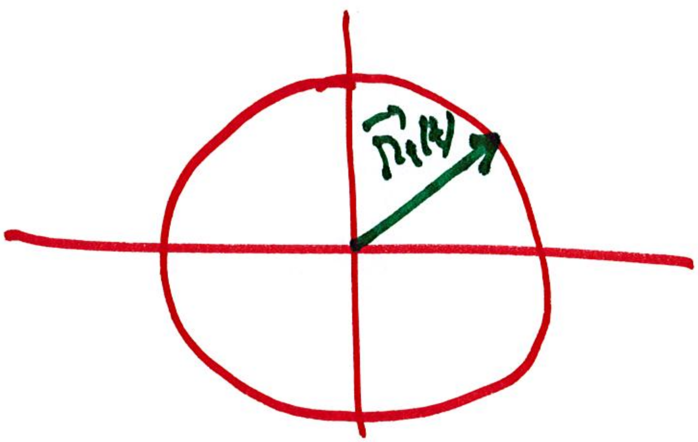

As $t$ varies $\vec{r}(t)$ describes the circle centered at $0$ with radius 1.



$$\vec{r}_1(t) = \langle \cos 2t, \sin 2t \rangle$$

$$x = \cos 2t, \quad y = \sin 2t$$

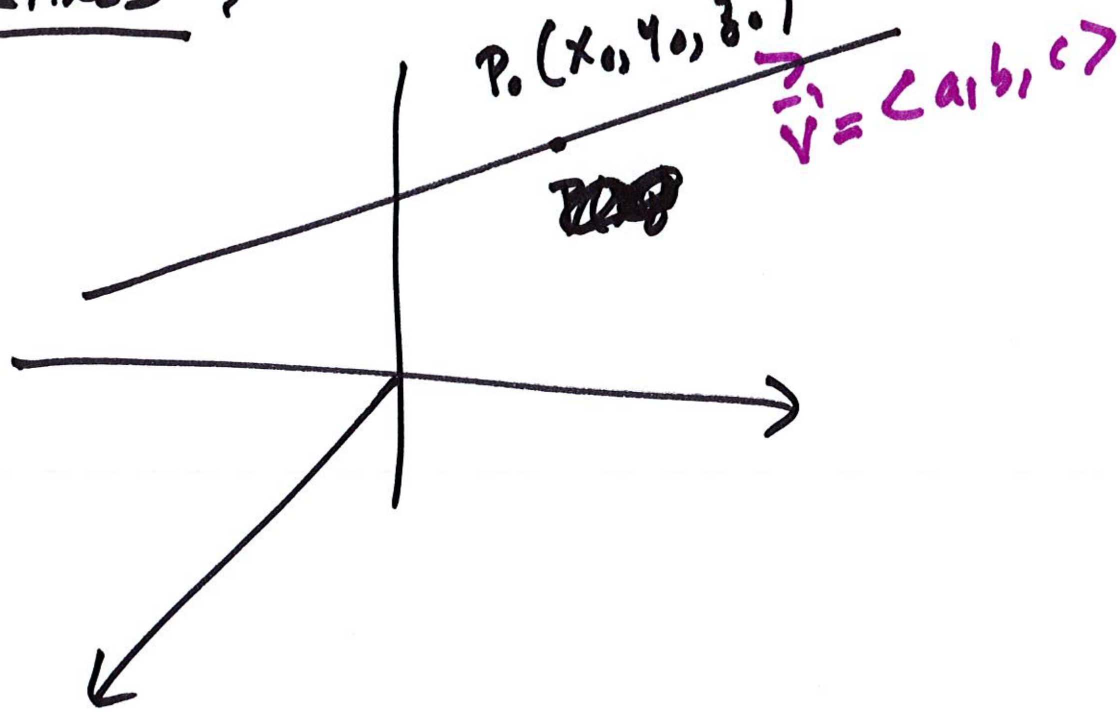
$$x^2 + y^2 = \cos^2 \underline{2t} + \sin^2 \underline{2t} = 1.$$




$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

Is called a vector function of $t$, $t$ in an interval. $[a, b]$

## Lines :




$$\vec{r}(t) = \langle \underbrace{x_0 + t}a,\ y_0 + tb,\ z_0 + tc \rangle$$

$$x = x_0 + ta,\quad y = y_0 + tb,\quad z = z_0 + tc$$

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

The domain of $\vec{r}(t)$ is the set of points (values of $t$) where it is defined.

Example:

$\vec{r}(t) = \langle \cos t, \sin t, t \rangle$

Domain of $\vec{r}(t)$ is $(-\infty, \infty)$

$\vec{r}(t) = \langle \sqrt{1-t}, \ln(1+t), t^2 \rangle$.

$\sqrt{1-t}$. $1-t \geq 0$, $1 \geq t$.

$$\boxed{t \leq 1}$$

$\ln(1+t)$; $1+t > 0$, $t > -1$

Conclusion: $-1 < t \leq 1$

Domain $(-1, 1]$

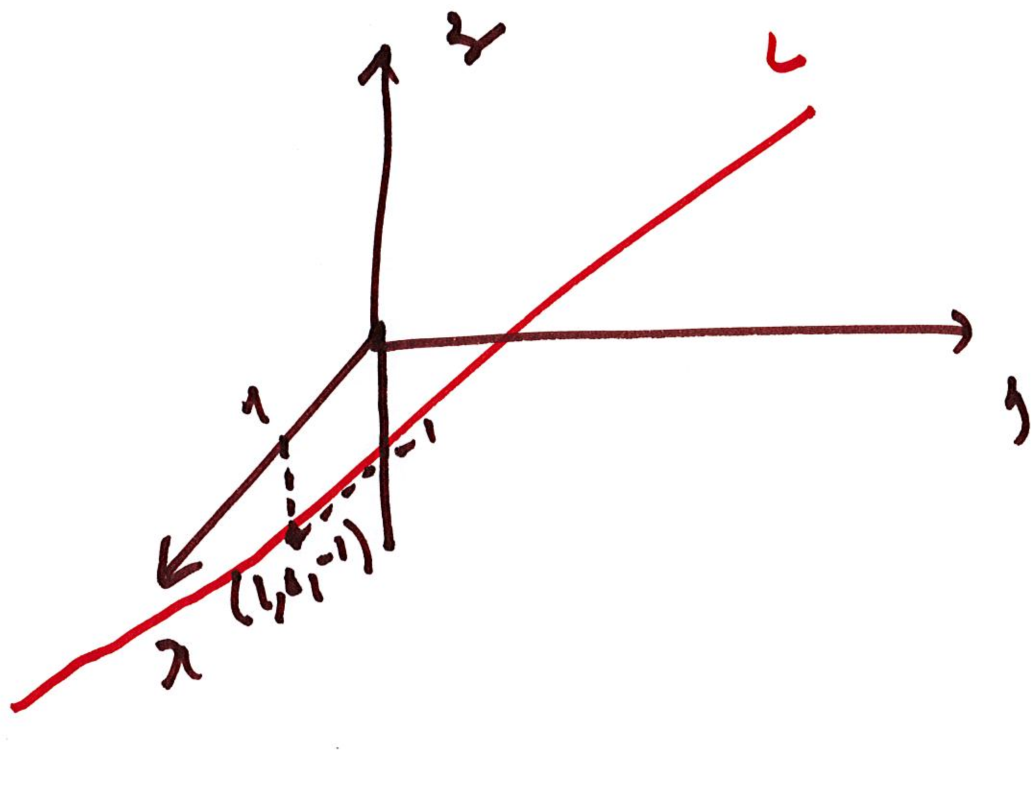
# Curve sketching.

$$\vec{r}(t) = \langle \underline{1} + 2t, \underline{0} + 3t, \underline{-1} + 4t \rangle$$

this is the vector equation
of a line passes through

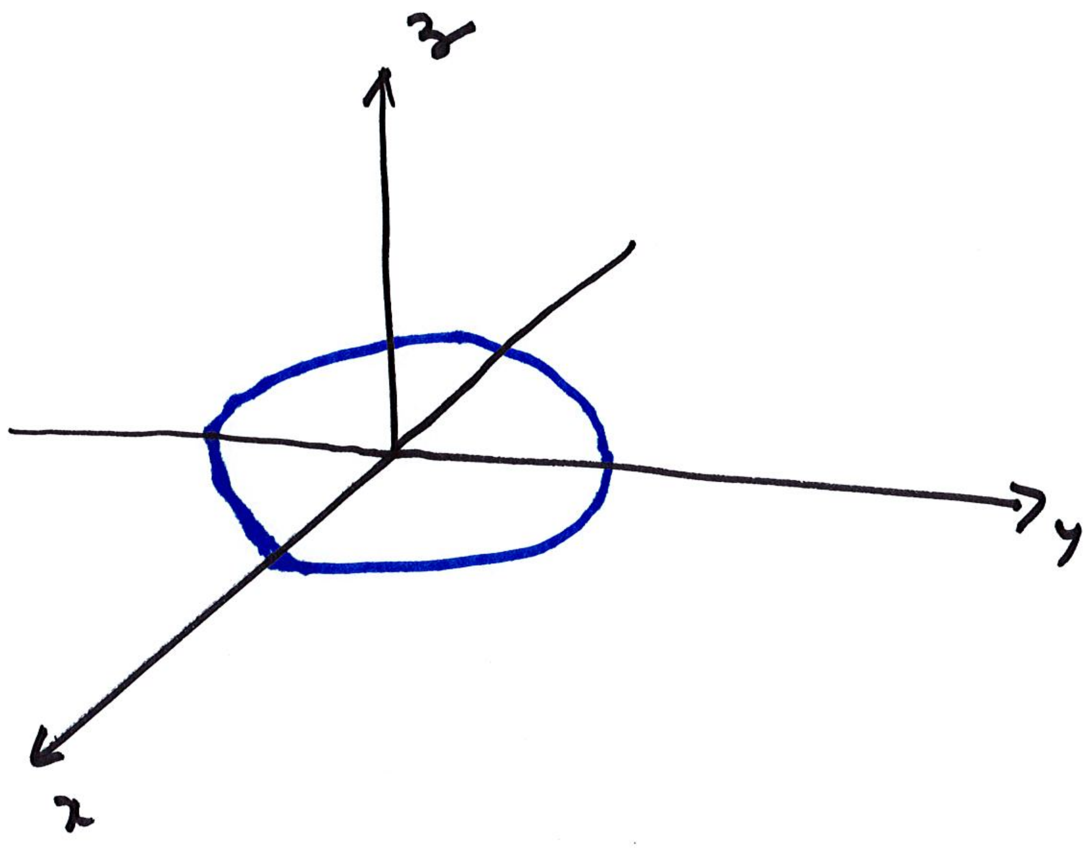
$P_0(1, 0, -1)$

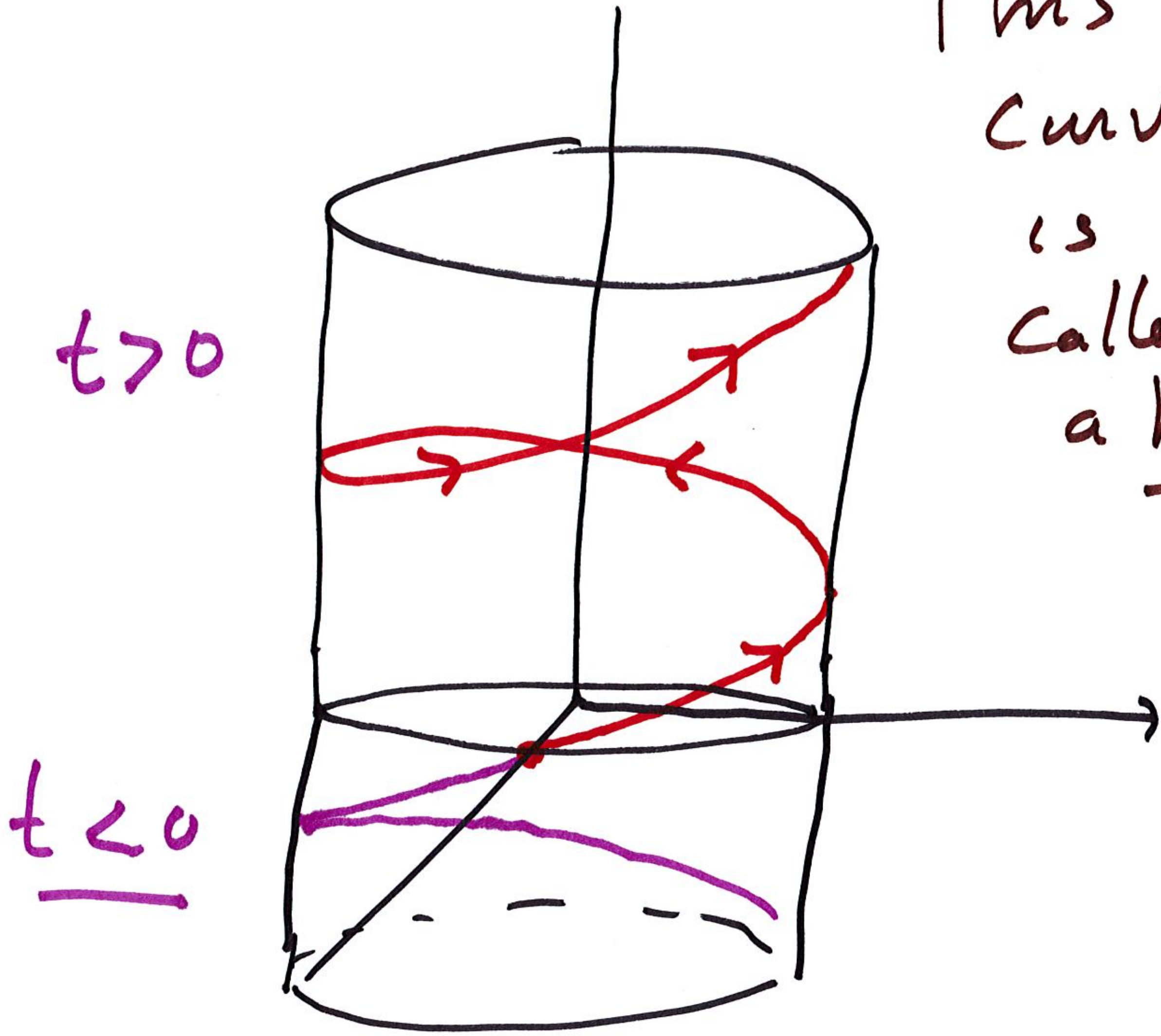
Parallel to $\vec{v} = \langle 2, 3, 4 \rangle$



$\vec{r}(t) = \langle \cos t, \sin t, t \rangle.$

$\vec{r}_0(t) = \langle \cos t, \sin t \rangle$






This curve is called a <u>helix</u>

$t=0, \quad \vec{r}(0) = \langle 1, 0, 0 \rangle$

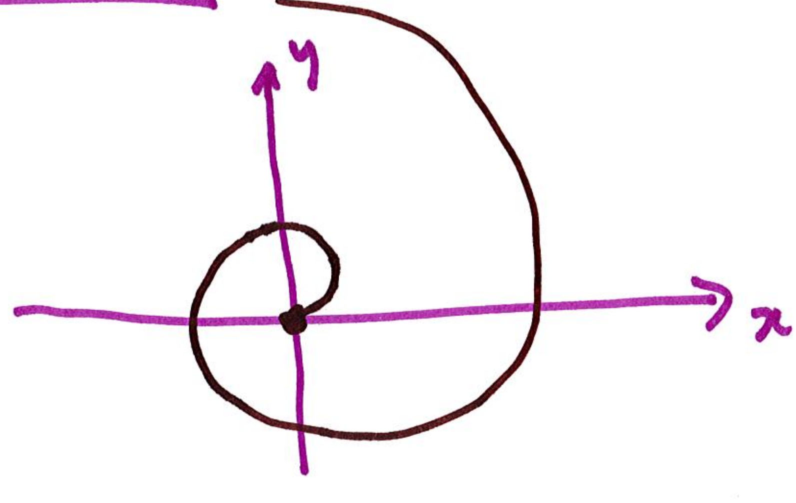
$\vec{r}(t) = \langle t\cos t, t\sin t, t \rangle.$

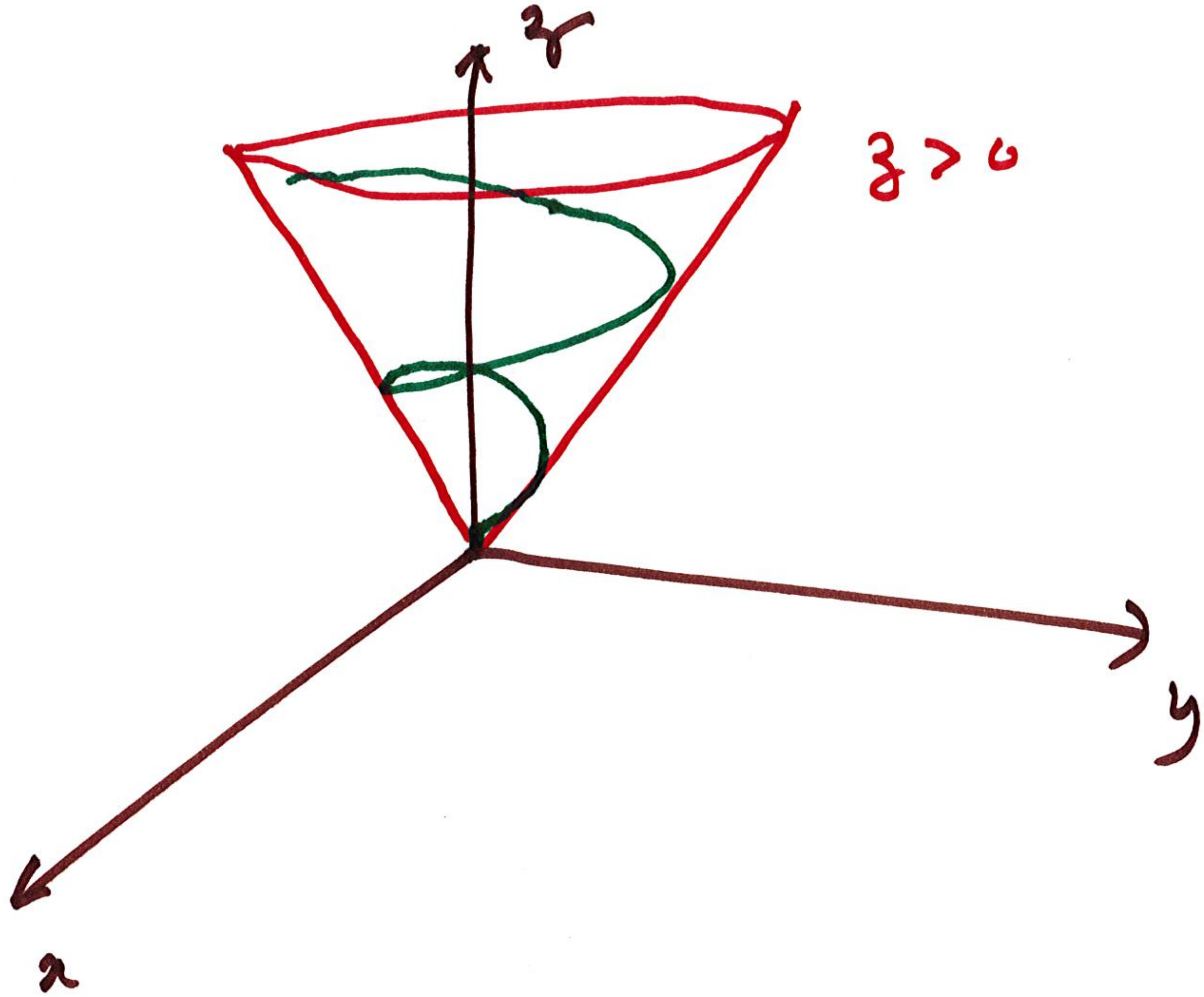
$x = t\cos t, \quad y = t\sin t$

$$x^2 + y^2 = t^2\cos^2 t + t^2\sin^2 t$$
$$= t^2(\underbrace{\cos^2 t + \sin^2 t}_{= 1})$$
$$= t^2$$

$$\boxed{x^2 + y^2 = t^2} \qquad t = z$$

Projection:



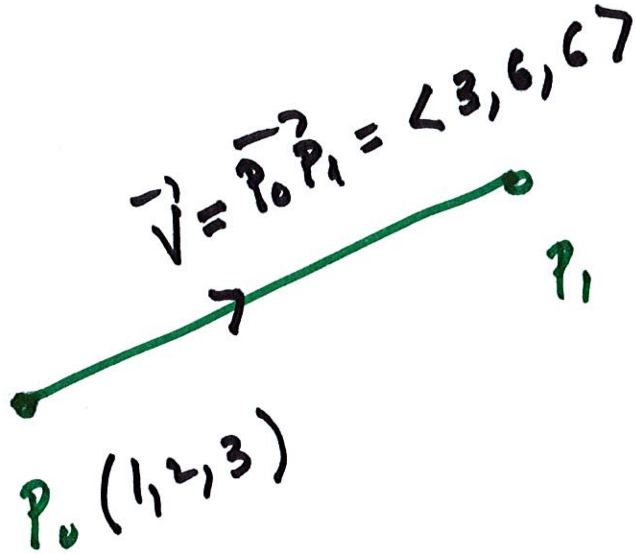



this curve lies on the surface

$$z^2 = x^2 + y^2.$$

# Finding Vector Equations for Curves.

<u>Example:</u> Segment of the line joining $P_0(1,2,3)$ and $P_1(4,8,9)$




$$\vec{r}(t) = \langle 1,2,3 \rangle + t\langle 3,6,6 \rangle.$$

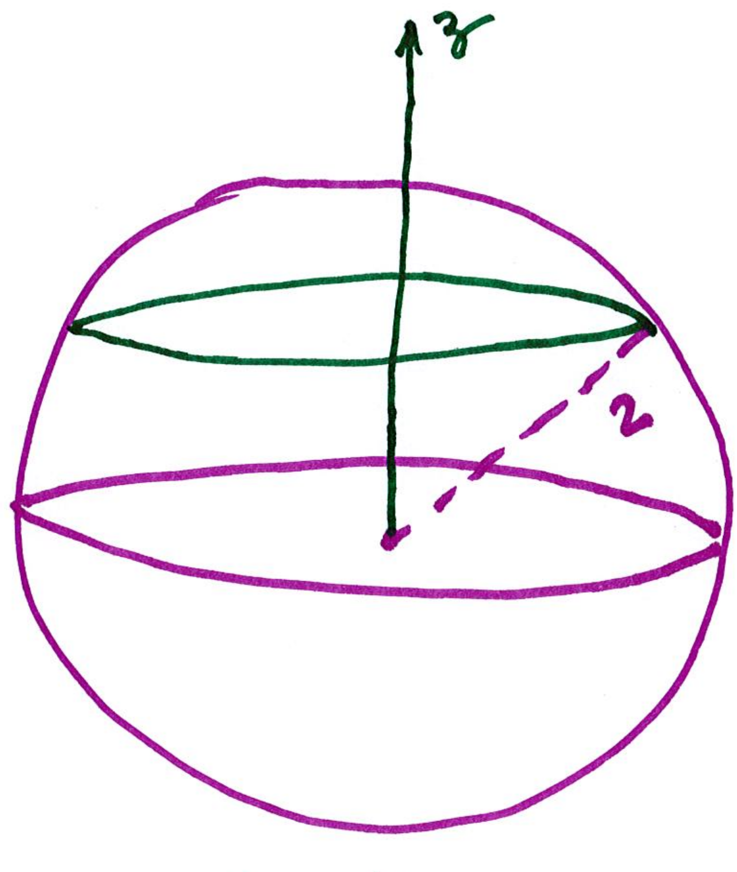
Restrict the values of $t$.

$\vec{r}(0) = \langle 1,2,3 \rangle$, $\vec{r}(1) = \langle 4,8,9 \rangle$

$t$ in $[0,1]$.

2) The curve given by the intersection of the sphere $x^2 + y^2 + z^2 = 4$

and the plane $z = 1$.




$$\begin{cases} x^2 + y^2 + z^2 = 4 \\ z = 1 \end{cases}$$

Replace $z=1$ in the first equation.

$$x^2+y^2+1=4$$

$$\begin{cases} x^2+y^2=3 \\ z=1. \end{cases}$$

$x=\sqrt{3}\cos t, \quad y=\sqrt{3}\sin t, \quad z=1$

$\vec{r}\,(t)=\langle \sqrt{3}\cos t, \ \sqrt{3}\sin t, \ 1 \rangle$

This <u>parametrizes</u> <u>the</u> <u>curve</u>.