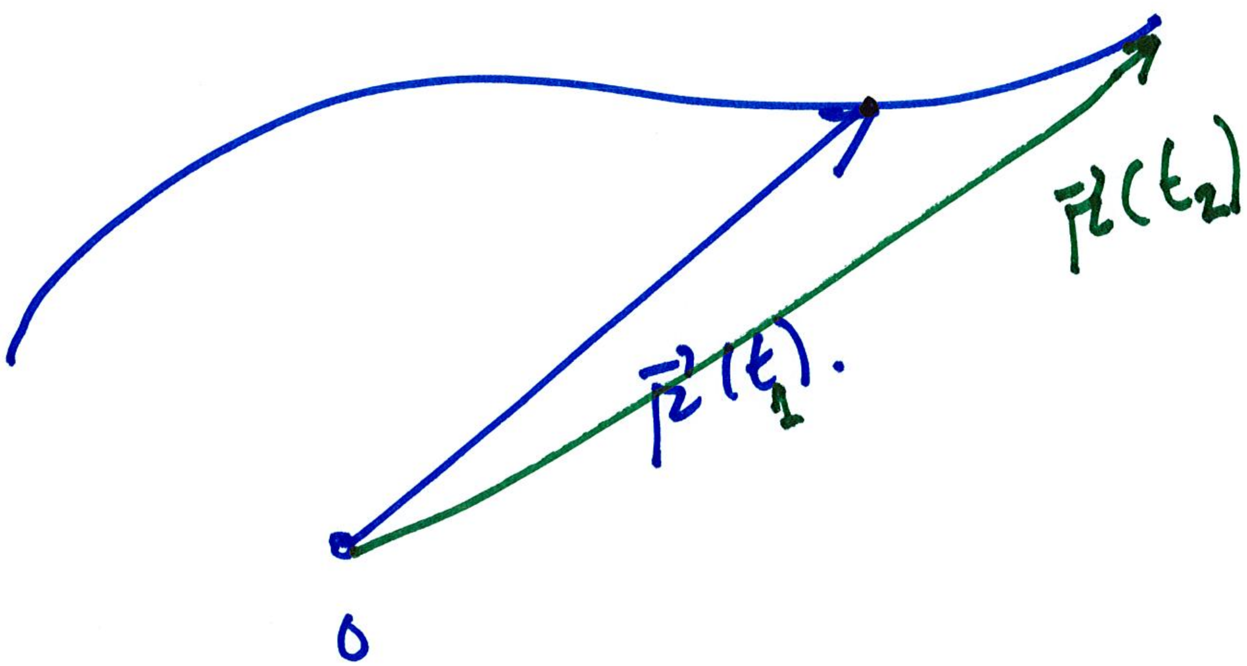
Lesson 6. Section 13.2

Derivatives and Integrals of Vector Functions.



Think of this in physical terms.

$\vec{r}(t)$ = Position of an object at time t.

Rate of change of $\vec{r}(t)$ with respect to t.

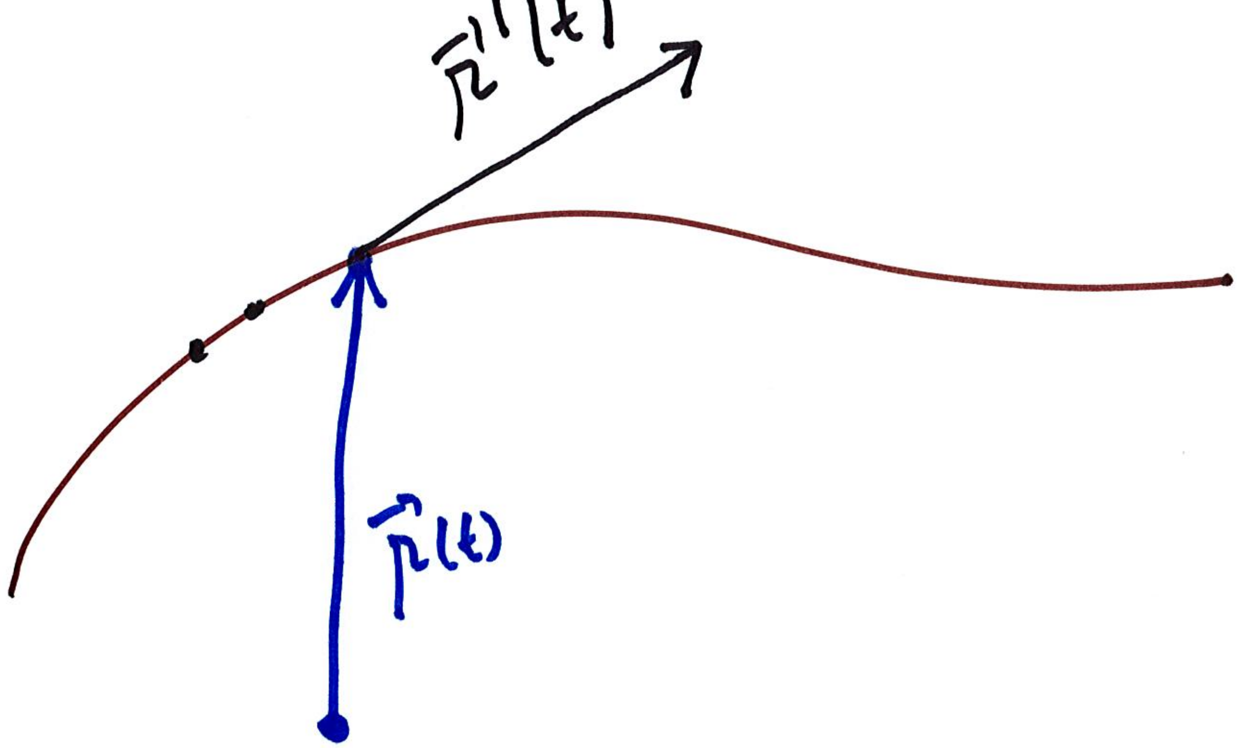
$$\frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

$$\frac{d}{dt}\vec{r}(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}.$$

This contains the following information:

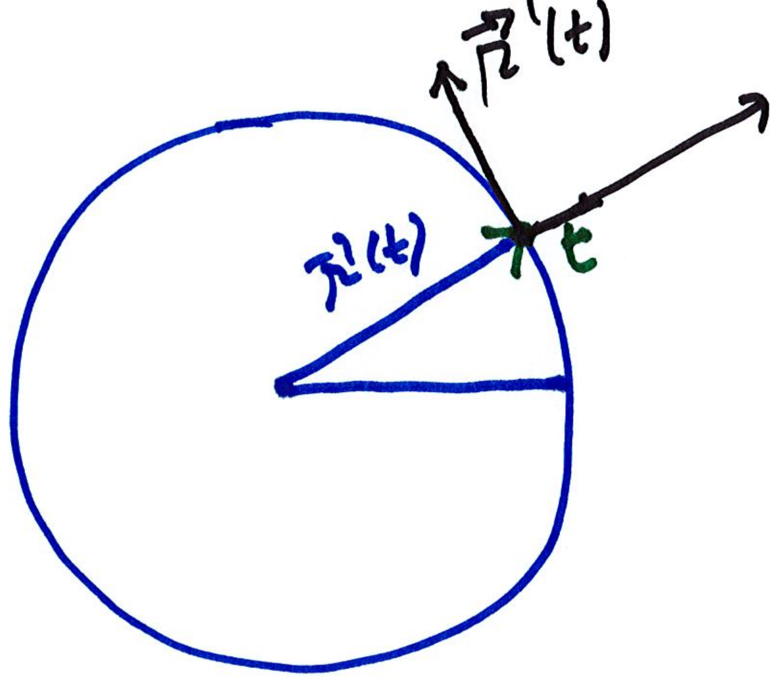
1) Speed ~~in~~ the ~~car~~ object is traveling

2) The change of direction.



$\vec{r}'(t) =$ Points in the direction of the tangent line to the curve described by $\vec{r}(t)$

or is parallel to the tangent line.



$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$
$$= f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}.$$

$$\vec{r}'(t) = \frac{d}{dt}\vec{r}(t) = f'(t)\vec{i} + g'(t)\vec{j} + h'(t)\vec{k}.$$

$$= \langle f'(t), g'(t), h'(t) \rangle.$$

$$\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}$$

$$= \left(\frac{f(t+\Delta t)-f(t)}{\Delta t}\right)\vec{i} + \left(\frac{g(t+\Delta t)-g(t)}{\Delta t}\right)\vec{j}$$

$$+ \left(\frac{h(t+\Delta t)-h(t)}{\Delta t}\right)\vec{k}.$$

$$\boxed{\frac{d}{dt}\vec{r}(t) = f'(t)\,\vec{i} + g'(t)\,\vec{j} + h'(t)\,\vec{k}}$$

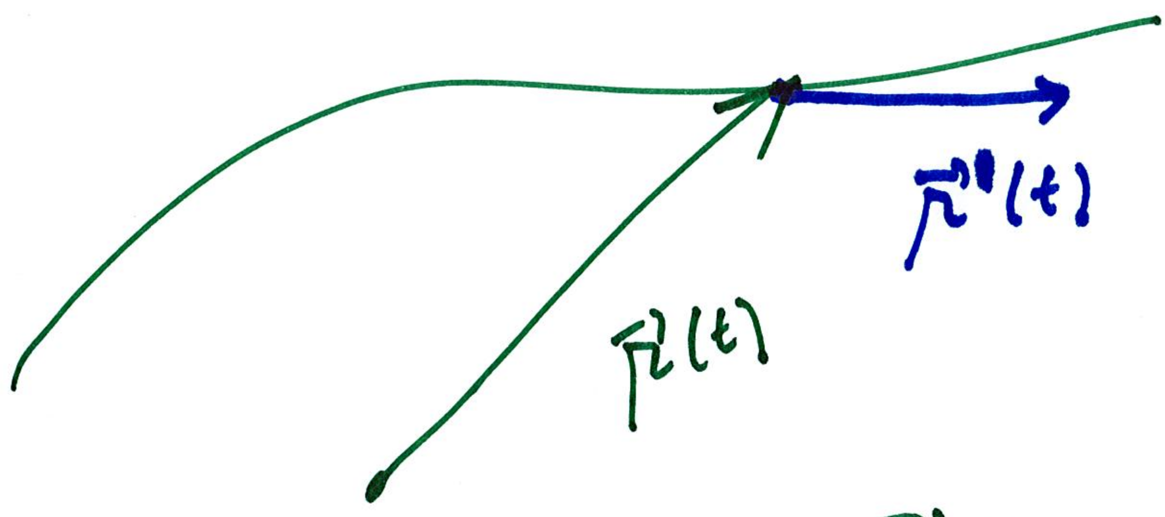
$\vec{r}\,'(t) = \frac{d}{dt}\vec{r}(t)$ = Velocity vector.

Speed . is a number

$$|\vec{r}\,'(t)|$$

$$|\vec{r}\,'(t)| = \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}.$$

The unit tangent vector



• A curve is given by $\vec{r}(t)$.

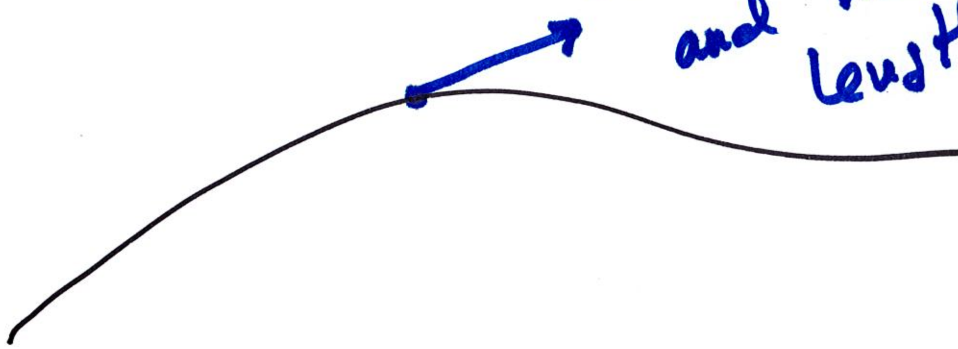
$\vec{r}'(t)$ is parallel to the line tangent to the curve.

<u>Unit vector:</u>

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$|\vec{r}'(t)| \neq 0.$$

$T(t)$ is tangent and has length $= 1$.



<u>Examples</u>: Find $\vec{r}'(t)$ and $\vec{T}(t)$.

1) $\vec{r}(t) = e^t\,\vec{i} + \cos t\,\vec{j} + t\sin t\,\vec{k}$.

$$\boxed{\vec{r}'(t) = e^t\,\vec{i} - \sin t\,\vec{j} + (\sin t + t\cos t)\,\vec{k}}$$

$$|\vec{r}'(t)| = \sqrt{e^{2t} + \sin^2 t + \sin^2 t + t^2\cos^2 t + 2t\sin t\cos t}.$$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}.$$

<u>$t = 0$</u>

At $t=0$

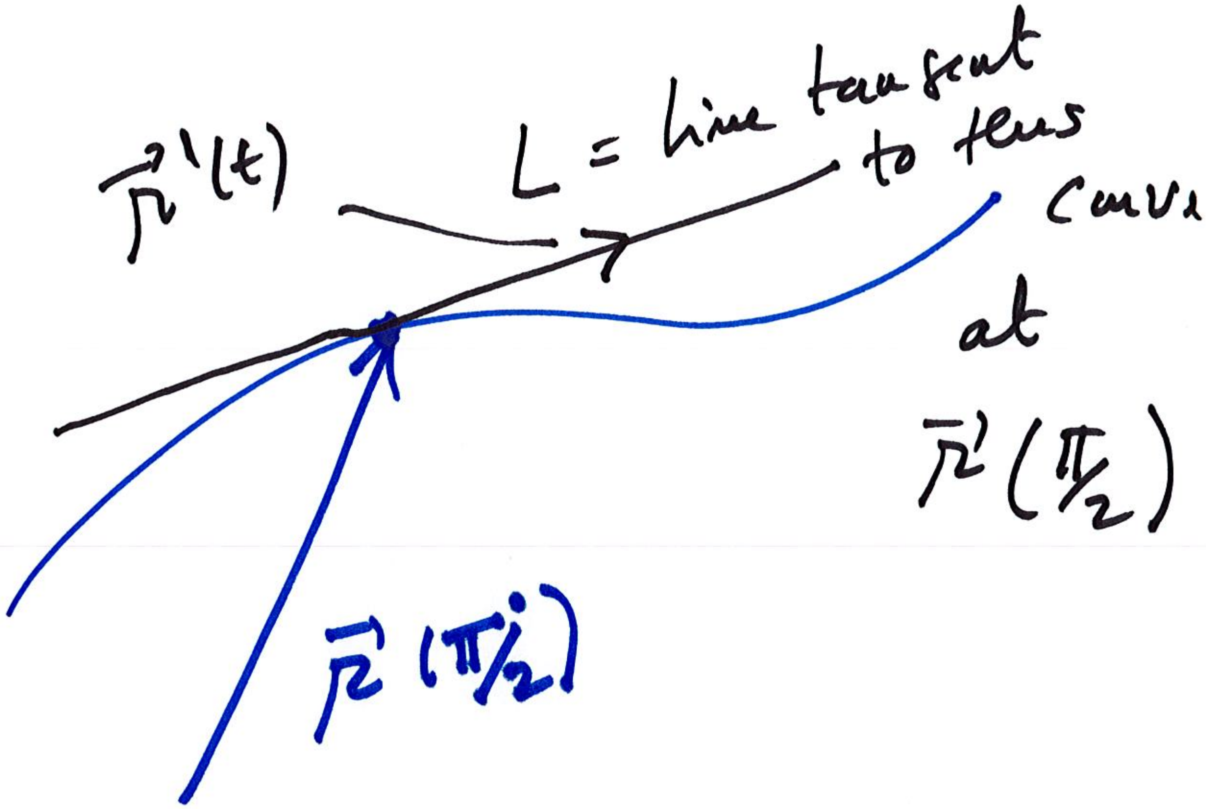
$$\boxed{\vec{r}'(0) = \vec{i}}$$

At $t=\pi/2$

$$\vec{r}'(\pi/2) = e^{\pi/2}\vec{i} - \vec{j} + \vec{k}$$

$$|\vec{r}'(\pi/2)| = \sqrt{e^{\pi} + 1 + 1} = \sqrt{2+e^{\pi}}.$$

$$\boxed{\vec{T}(t) = \frac{1}{\sqrt{2+e^{\pi}}}\left(e^{\pi/2}\vec{i} - \vec{j} + \vec{k}\right)}$$



The line L passes through

$$\vec{r}(\pi/2) = e^{\pi/2}\,\vec{i} + \frac{\pi}{2}\,\vec{k}.$$

$$P_0\left(e^{\pi/2},\, 0,\, \frac{\pi}{2}\right)$$

$\vec{r}\,'(\pi/2)$ = Parallel to L.

$$\vec{r}\,'(\pi/2) = e^{\pi/2}\,\vec{i} - \vec{j} + \vec{k}.$$

Equation for the line

L:

$$\vec{u}(t) = \langle e^{\pi/2}, 0, \pi/2 \rangle$$

$$+ \; t \langle e^{\pi/2}, -1, 1 \rangle$$

Rules of Differentiation

1) $\vec{r}(t)$ and $\vec{v}(t)$

$$\frac{d}{dt}\left(\vec{r}(t) + \vec{v}(t)\right) = \frac{d\vec{r}}{dt} + \frac{d\vec{v}}{dt}.$$

2) $$\frac{d}{dt}\left(\vec{r}(t) \cdot \vec{v}(t)\right) =$$

$$\vec{r}\,'(t) \cdot \vec{v}(t) + \vec{r}(t) \cdot \vec{v}\,'(t)$$

3) $$\frac{d}{dt}\left(\vec{r}(t) \times \vec{v}(t)\right) =$$

$$\vec{r}\,'(t) \times \vec{v}(t)$$

$$+ \vec{r}(t) \times \vec{v}\,'(t).$$

Example:

$$\vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + t\,\vec{k}$$

$$\vec{V}(t) = t\,\vec{i} + t^2\,\vec{j} + t^3\,\vec{k}.$$

$$\vec{r}(t)\cdot\vec{V}(t) = \underbrace{t\cos t} + t^2\sin t + t \cdot t^3$$

$$\frac{d}{dt}\,\vec{r}(t)\cdot\vec{V}(t) = \cos t - t\sin t$$

$$+ 2t\sin t + t^2\cos t + 4t^3.$$

$$= \underline{\cos t + t\sin t + t^2\cos t + 4t^3}$$

$$\vec{r}'(t) = -\sin t\,\vec{i} + \cos t\,\vec{j} + \vec{k}$$

$$\vec{v}'(t) = \vec{i} + 2t\,\vec{j} + 3t^2\,\vec{k}$$

$$\vec{r}'(t) \cdot \vec{v}(t) + \vec{r}(t) \cdot \vec{v}'(t)$$

$$=$$