Lesson 9 Section 14.1

Functions of Several Variables.

A function of the variables $(x_1, x_2, x_3, \ldots, x_n)$ is a rule that associates $(x_1, x_2, \ldots, x_n)$ to a number $f(x_1, x_2, \ldots, x_n)$

Examples:

$$f(x_1, x_2) = \frac{x_1 x_2}{x_1^2 + x_2^2}$$

$x_1 = x, \quad x_2 = y$

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

$$f(x, y) = \ln\left(\frac{x}{y}\right)$$

The domain of a function is the set of points where f is defined.

$$f(x,y) = \frac{xy}{x^2+y^2}.$$

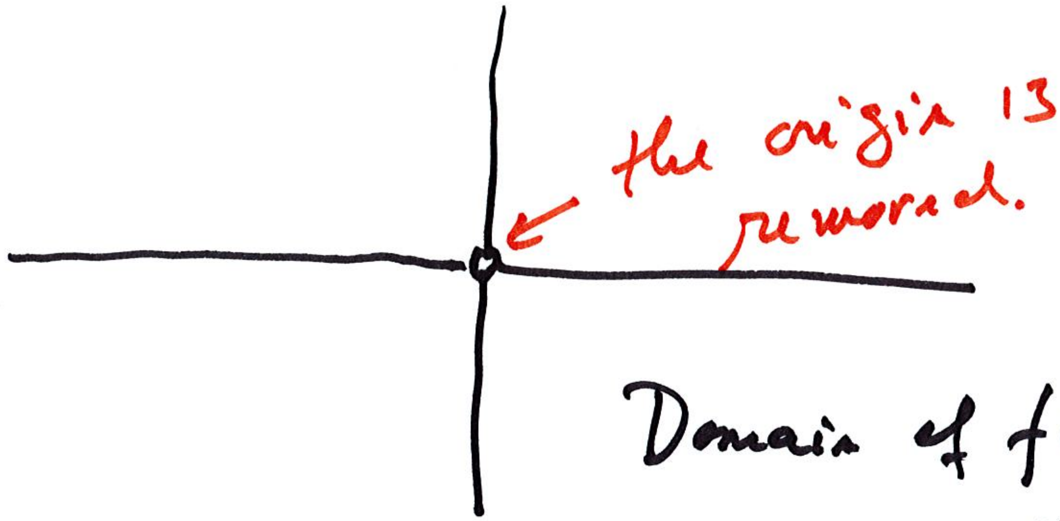
What is the domain of $f(x,y)$?

We have to make sure the denominator is not $= 0$.

$$x^2+y^2 \neq 0$$

there is no other restriction.

The domain of $f(x,y)$ is the set of (x,y) such that $(x,y) \neq 0$.



Domain of $f(x,y) = \dfrac{xy}{x^2+y^2}$.

$$f(x,y) = \ln\left(\frac{x}{y}\right).$$

(1) $y \neq 0$

(2) Recall that $\ln(t)$ is only defined for $t > 0$.

So we must have

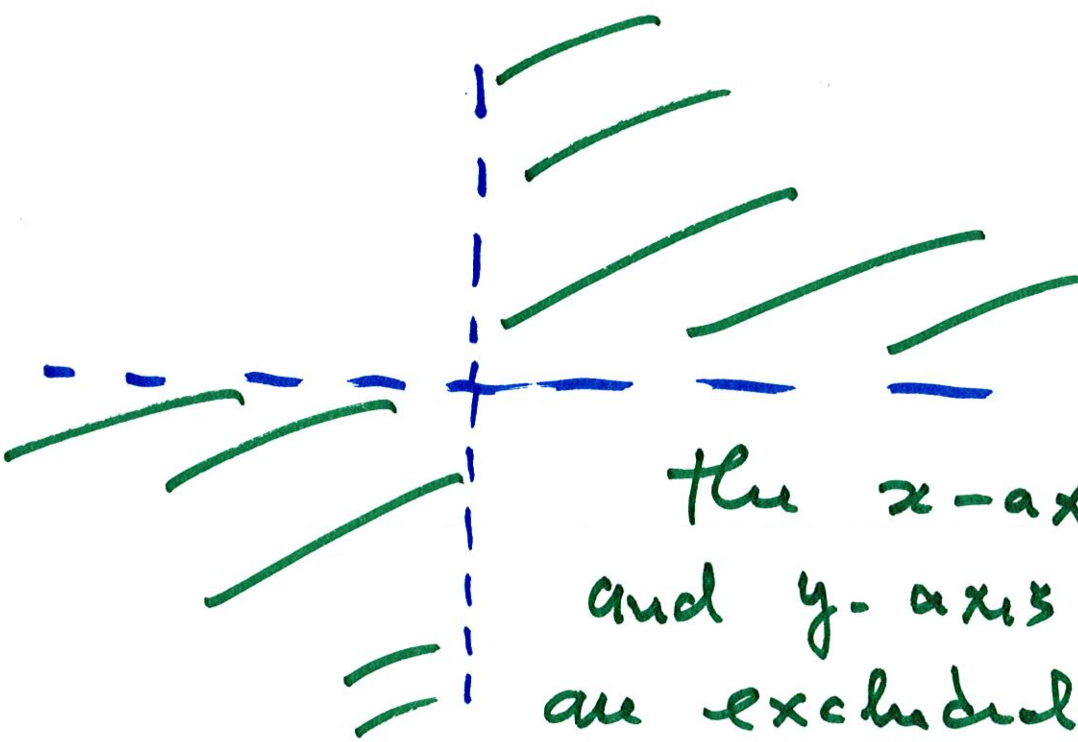
$$x/y > 0$$

this holds if either

$x > 0$ and $y > 0$

or

$x < 0$ and $y < 0$



the x-axis and y-axis are excluded.

$$f(x,y) = \sqrt{1-x^2} + \sqrt{y^2-1}.$$

We must have

$$1-x^2 \geqslant 0$$

and $\quad y^2-1 \geqslant 0$

$$\begin{cases} 1-x^2 \geqslant 0 \;,\; 1 \geqslant x^2 \;; \\ -1 \leq x \leq 1 \end{cases}$$

$$y^2-1 \geqslant 0$$

$$\begin{cases} y^2 \geqslant 1. \\ y \geqslant 1 \quad \text{or} \quad y \leq -1. \end{cases}$$

Domain of $f(x,y)$

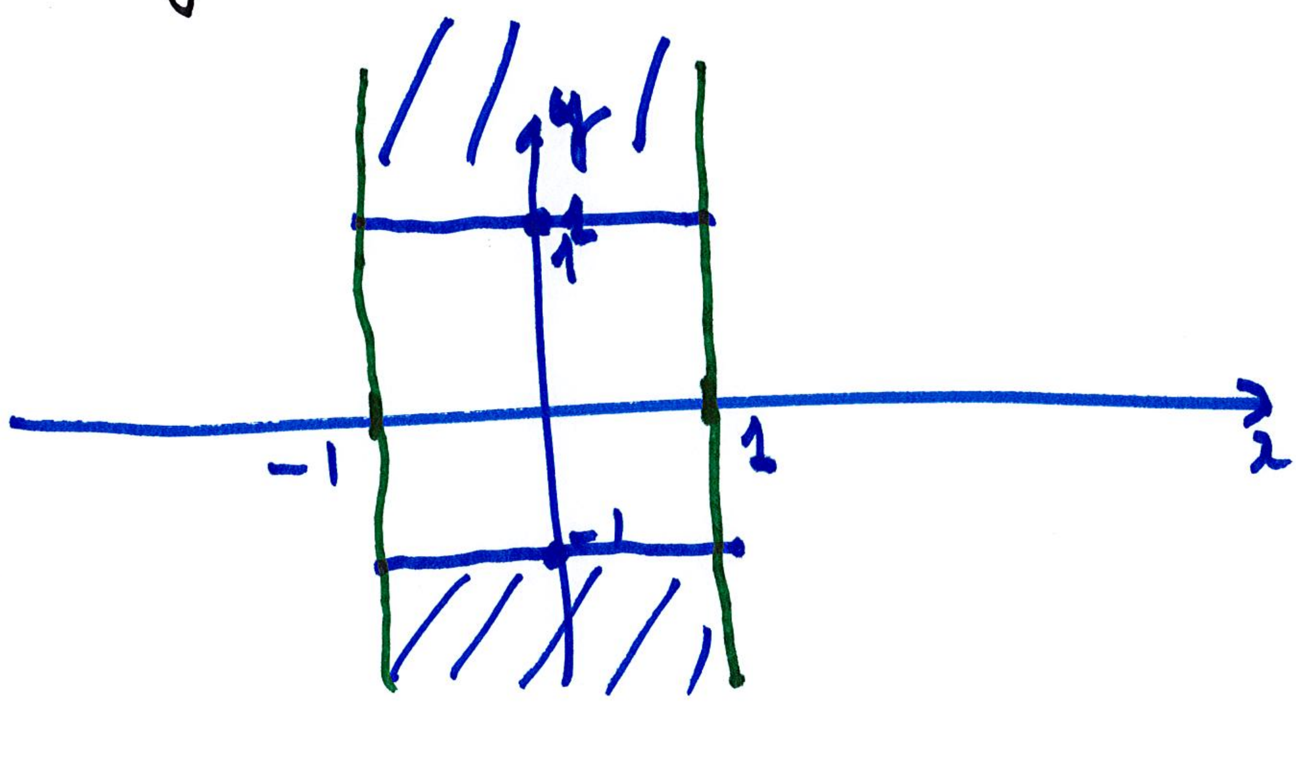
is the set of

(x,y) such that

$$-1 \le x \le 1$$

and

$$y \ge 1 \text{ or } y \le -1$$



The Range of $f(x, y)$

is the set of points z which are of the form $z = f(x, y)$.

These are the values achieved by $f(x, y)$.

Example:

$$f(x) = \sin x$$

$$z = \sin x$$

the range is $[-1, 1]$

any number z in $[-1, 1]$

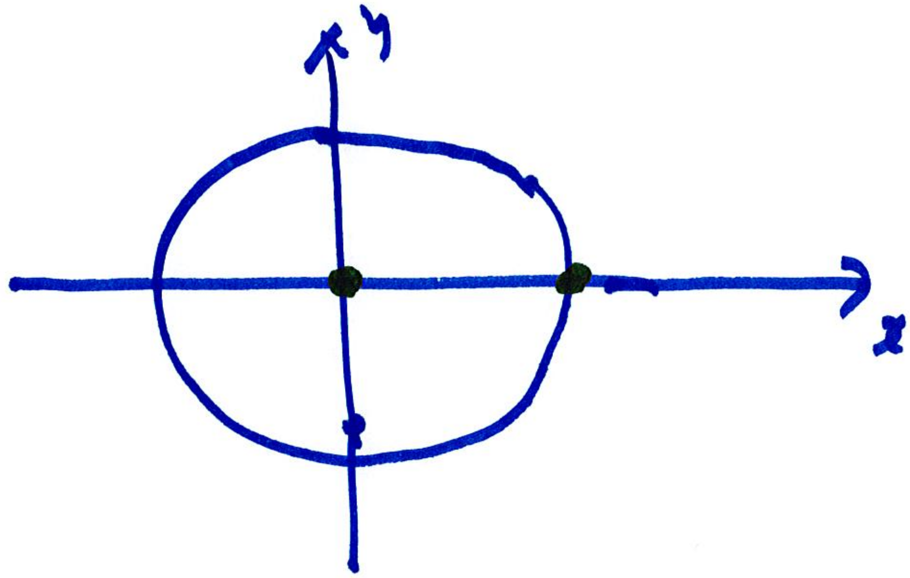
is of the form

$$z = \sin x \quad \text{for some } x.$$

$$f(x, y) = \sqrt{1 - x^2 - y^2}.$$

Domain. $1 - x^2 - y^2 \geq 0$

$$x^2 + y^2 \leq 1.$$



$$0 \leq 1 - x^2 - y^2 \leq 1$$

$$0 \leq \sqrt{1 - x^2 - y^2} \leq 1$$

when $x = y = 0$

$$f(x,y) = \sqrt{1-x^2-y^2} = 1.$$

when $x = 0,\ y = 1$

$$f(x,y) = \sqrt{1-0-1} = 0.$$

The range of $f(x,y)$ is

$[0,1]$.

Graphs of functions of two variables.

the graph of $f(x,y)$ is the set

(x, y, z) such that

$$\boxed{z = f(x, y)}$$

The case of 1-dimension

$f(x) = x^2$

the graph of $f(x)$ is the set (x, y) such that $y = x^2$.

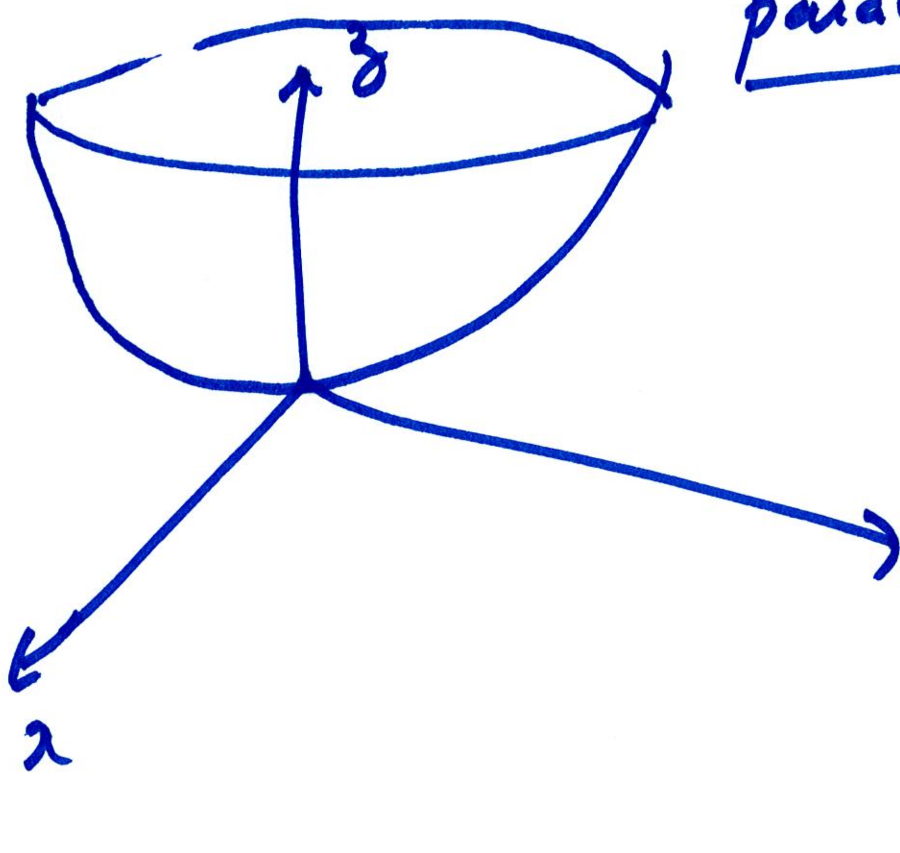
What is the graph of

$$f(x,y) = x^2 + y^2 \, ?$$

Set (x, y, z) such that

$$z = x^2 + y^2.$$

paraboloid



Graph of

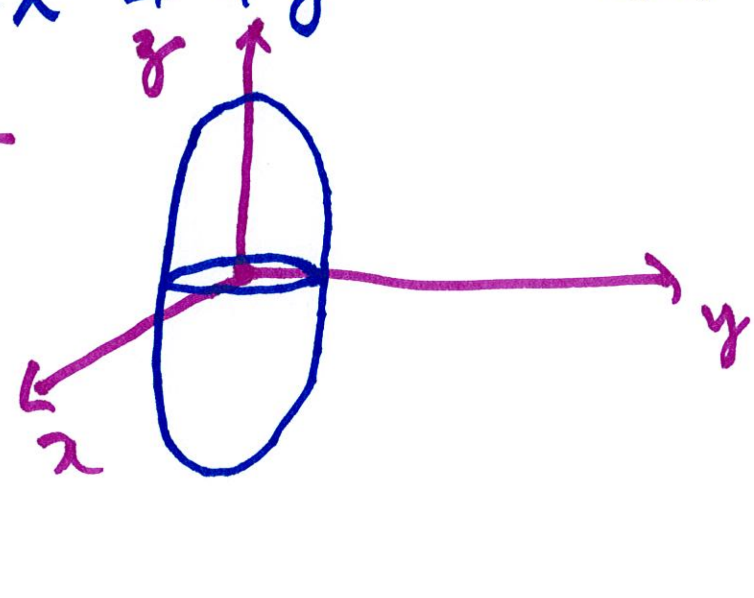
$$f(x,y) = \sqrt{1-2x^2-4y^2}.$$

Step 1: $f(x,y) \geq 0$

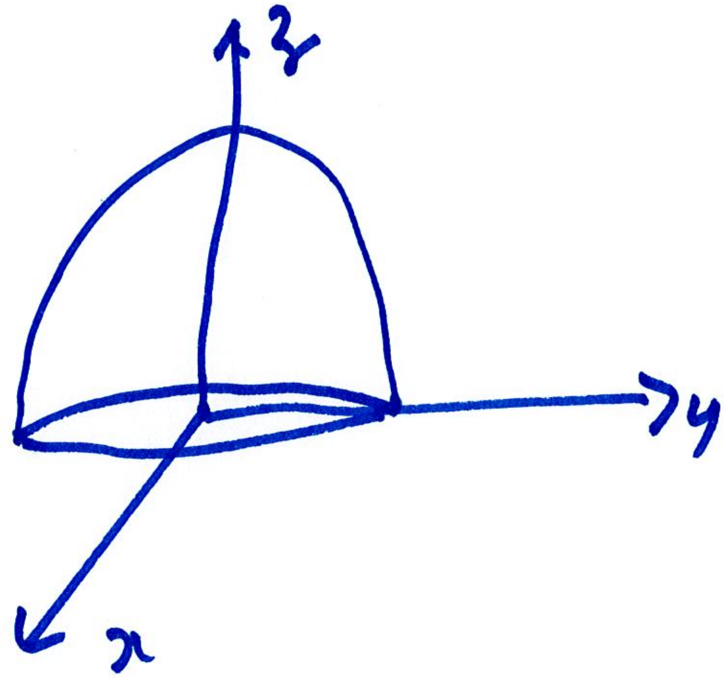
Step 2: $z = \sqrt{1-2x^2-4y^2} \geq 0$

$$z^2 = 1 - 2x^2 - 4y^2$$

$$\boxed{z^2 + 2x^2 + 4y^2 = 1.}$$



Graph of $f(x,y) = \sqrt{1-2x^2-4y^2}$

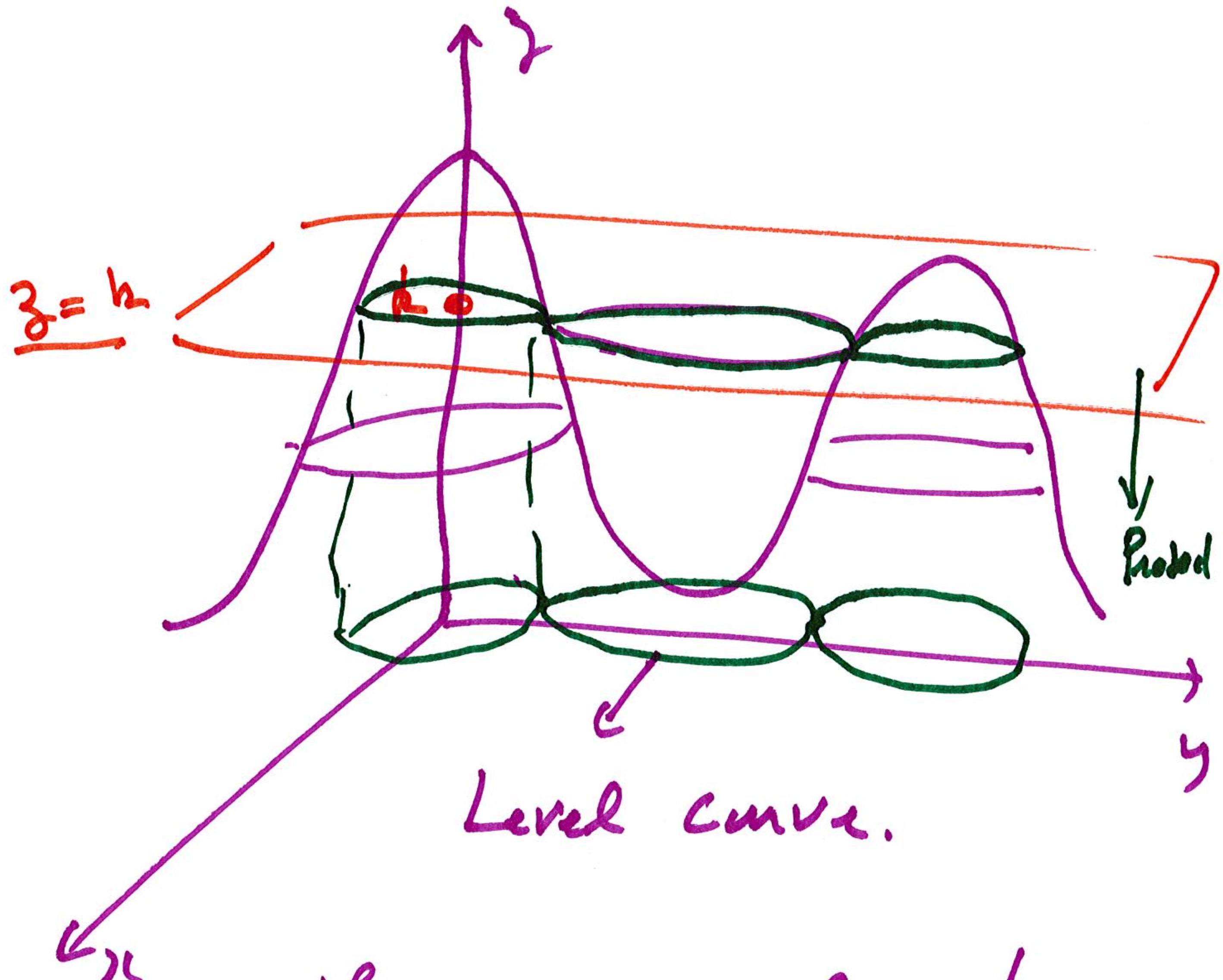


Level Curves

The level curves of a function

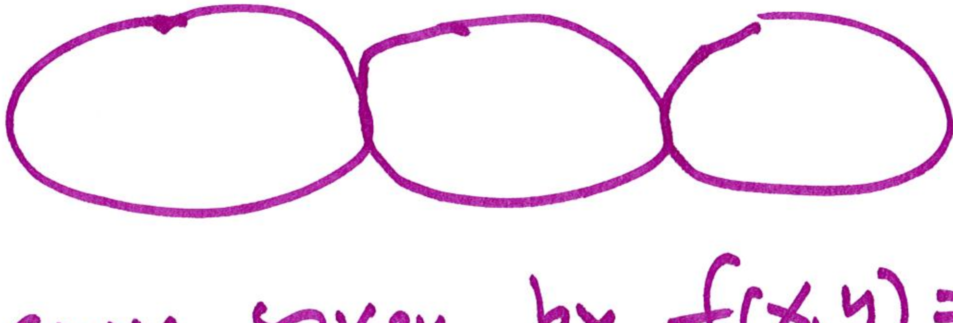
$$f(x,y)$$

are curves given by $f(x,y) = k$ where k is a constant.



this corresponds to

the ~~values~~



the curve given by $f(x, y) = k$.

Level curves of

$$f(x,y) = \sqrt{y^2 - x^2}.$$

Level curves $f(x,y) = k$.

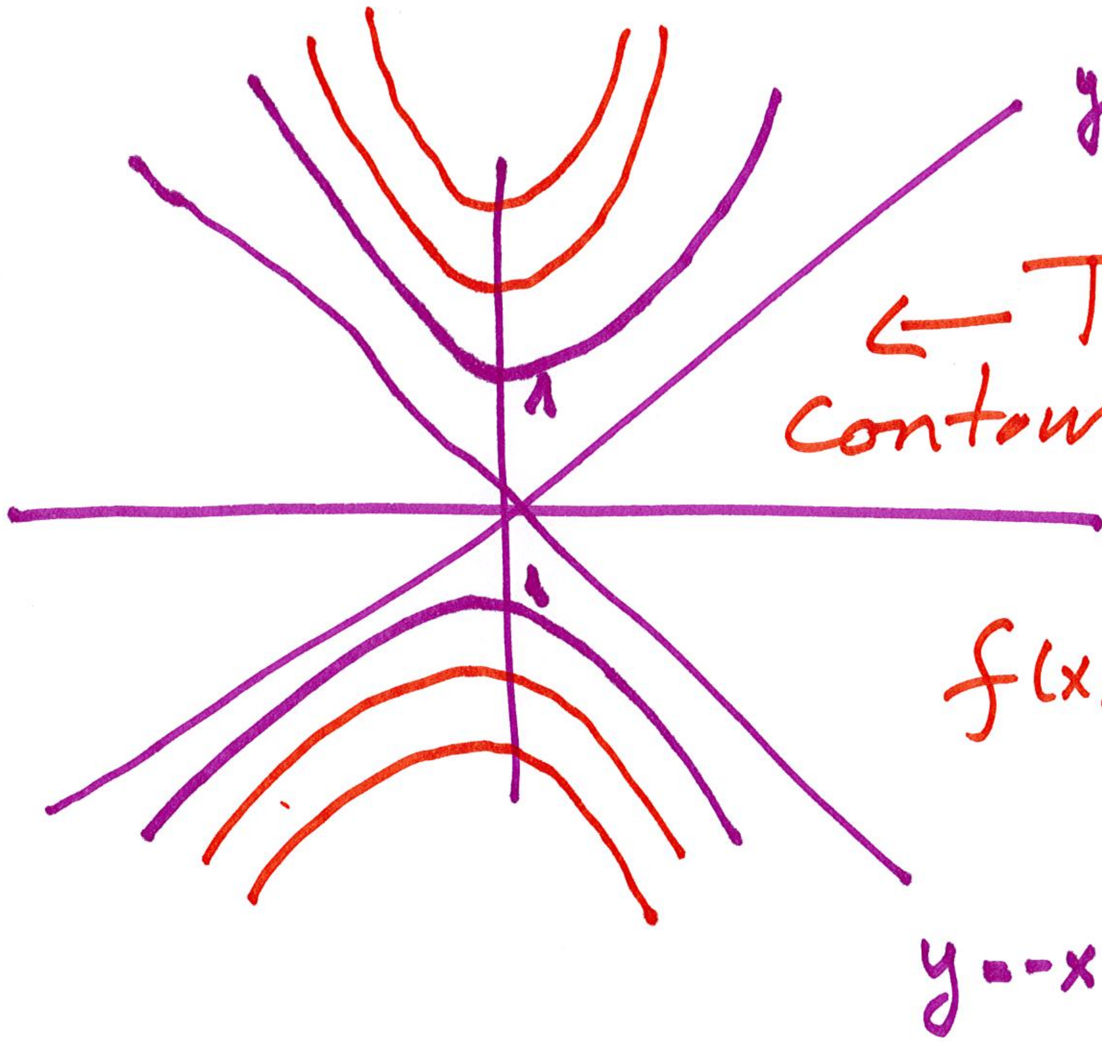
$$\sqrt{y^2 - x^2} = k.$$

$k \geq 0$.

$k = 0$: $\sqrt{y^2 - x^2} = 0$

$$y^2 = x^2.$$

$$y = x \text{ or } y = -x.$$



← The contour map of $f(x,y)$.

$\underline{k=1}$: $\quad y^2 - x^2 = 1$

$k \geq 0 \quad y^2 - x^2 = k^2$