MA 262

Office: MA 640

Hours: Th: 1:00 - 2:00 ; Wed 1:00 - 2:00

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Course Info.

WWW.MATH.PURDUE.EDU/MA 262

for the Webassign link.

WWW.MATH.PURDUE.EDU/NSABARRE/LADE.HTML

Course Calendar, Lecture Notes.

Lesson 1. A Differential Equation is an equation involving one or more derivatives of an unknown function.

Ex:

$$\frac{d^2 y(x)}{dx^2} + y(x) = 0$$

$$\tan\left(\frac{dy}{dx}\right) + \frac{d^3 y}{dx^3} = e^x.$$

The function:

$$y(x) = A \sin x + B \cos x$$

satisfy

$$y^{(2)}(x) + y(x) = 0$$

$$\boxed{y^{(2)} = \frac{d^2 y}{dx^2} \; ; \; y^{(n)} = \frac{d^n y}{dx^n}}$$ notation.

$$y'(x) = A \cos x - B \sin x$$

$$y^{(2)}(x) = - A \sin x - B \cos x$$

$$y^{(2)}(x) + y(x) = 0$$

This is called the general solution to

$$\frac{d^2 y}{dX^2} + y(x) = 0$$

To determine A, B we need additional conditions: Example. $y(0) = 1; \; y'(0) = 2.$

$y(x) = A\sin x + B\cos x$

$y(0) = B = 1$

$y'(x) = A\cos x - B\sin x$; $y'(0) = A = 2.$

So $y(x) = 2\sin x + \cos x$

Satisfies
$$\begin{cases} \dfrac{d^2y}{dx^2} + y(x) = 0 \\ y(0) = 1, \; y'(0) = 2 \end{cases}$$

This is called an <u>initial value problem</u>. because we are given "initial conditions" at $x=0$.

<u>Ex:</u> $\dfrac{dy}{dx} = m\,y(x)$; $y(x) = Ae^{mx}$ general solution

$$\begin{cases} \dfrac{dy}{dx} = m\,y(x) \\ y(0) = 1 \end{cases}$$

$$\boxed{y(x) = e^{mx}}$$

Equation of 2nd order we need two initial conditions, 1st order we need only one initial condition.

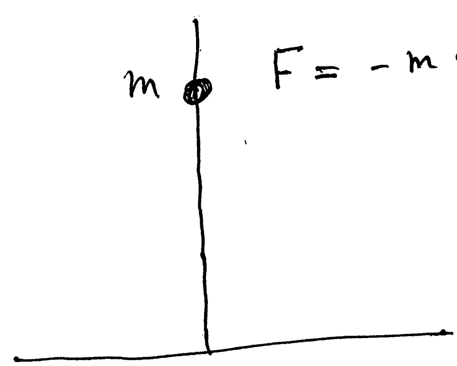
The order of a differential equation is the order of the highest derivative it contains.

Newton's Law of motion: $F = ma$

$y(t) =$ position at time t.

$$a = y''(t) = \frac{d^2y}{dt^2}.$$

$$\boxed{m\frac{d^2y}{dt^2} = F.}$$

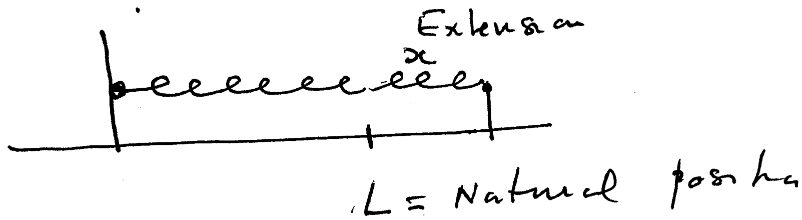


Eg: Gravitational force

$$m\frac{d^2y}{dt^2} = -mg; \quad \frac{d^2y}{dt^2} = -g.$$

$$\boxed{y(t) = -\frac{1}{2}gt^2 + At + B}$$

Hooke's Law:



$$F = -kx$$

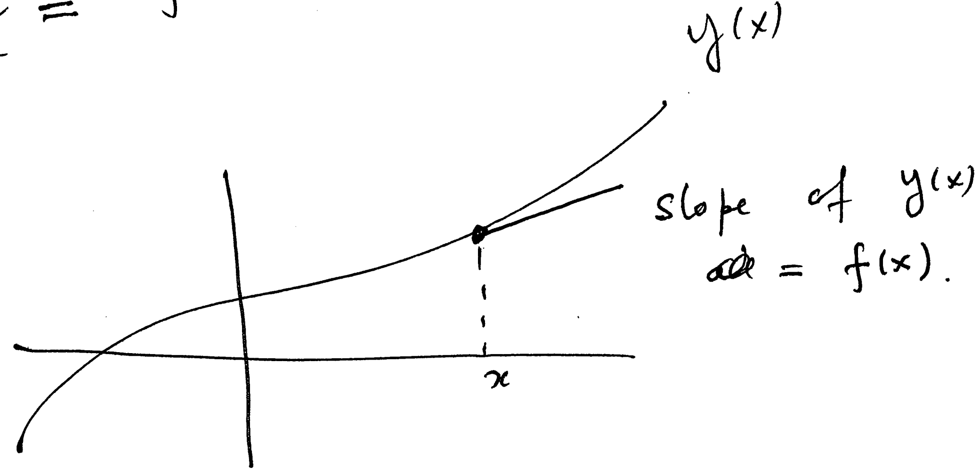
$$m \frac{d^2 x(x)}{dt} = -kx.$$

$$\frac{d^2 x(t)}{dt^2} + \frac{k}{m} x = 0$$

$$\boxed{x(t) = A \cos\sqrt{\tfrac{k}{m}}\, t + B \sin\sqrt{\tfrac{k}{m}}\, t}$$

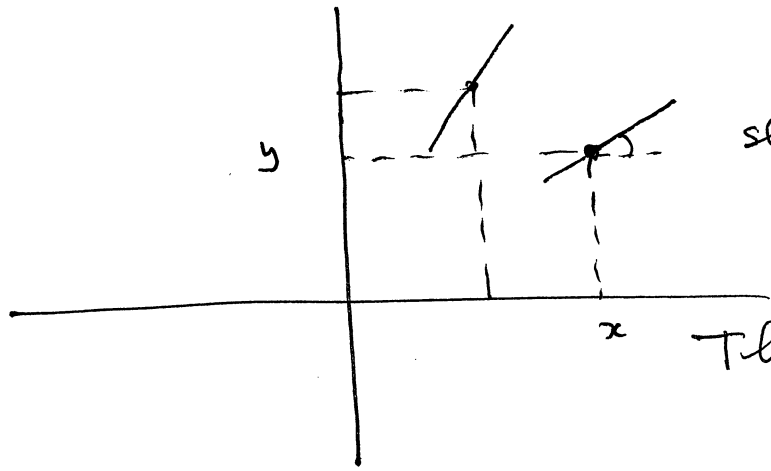
The Geometry of 1st order Diff. Eqs

$$\frac{dy}{dx} = f(x).$$



Direction Field for

$$\frac{dy}{dx} = f(x,y)$$

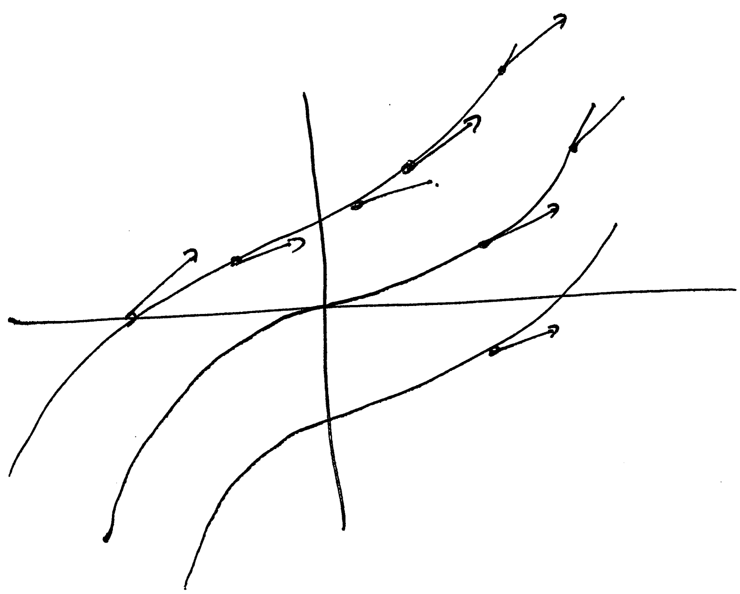


for each pt (x,y) draw a line with slope $= f(x,y)$.

The graph of $y(x)$ is tangent to these lines.

Ex: $\frac{dy}{dx} = 3x^2$; general Sol

$$y(x) = x^3 + c.$$



~~Unsquared~~ Given the family of curves:

$$x^2 + y^2 = 2cx$$

Determine the differential Equation satisfied by $y(x)$ by giving the slope of the tangent line at a point (x, y).

In other words: determine $\frac{dy}{dx}$

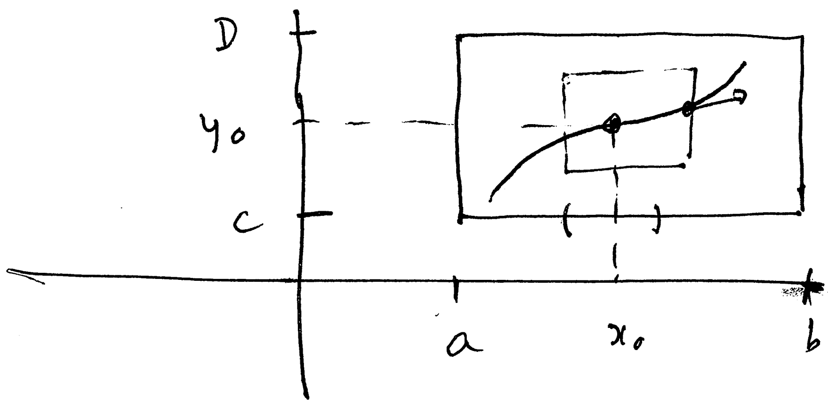
$$2x + 2y\frac{dy}{dx} = 2c.$$

$$2y\frac{dy}{dx} = 2c - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{c - x}{y}}$$

Existence and Uniqueness.

$$\frac{dy}{dx} = f(x, y) \quad ; \quad y(x_0) = y_0$$



Given $f(x,y)$ find a curve passing thru (x_0, y_0) having slope $f(x,y)$

$$R = \{ (x,y) : \quad a \le x \le b ; \quad c \le y \le d \}$$

If $\frac{\partial f}{\partial y}$ is continuous on R

then there exists a unique solution

$y(x)$ to $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$

on an interval I containing x_0

Examples:
$$\begin{cases} \dfrac{dy}{dx} = y^2 \\ y(1) = 1. \end{cases}$$

$$y(x) = \frac{1}{2-x} \quad ; \quad \frac{dy}{dx} = \frac{1}{(2-x)^2}$$

$$y(1) = 1.$$

$$f(x,y) = y^2 \quad , \quad \frac{\partial f}{\partial y} = 2y \quad \text{continuous.}$$

However $y(x)$ is defined only on

$$x < 2$$

Example:
$$\begin{cases} \dfrac{dy}{dx} = \frac{3}{2}\, y^{1/3} \\ y(0) = 0 \end{cases}$$

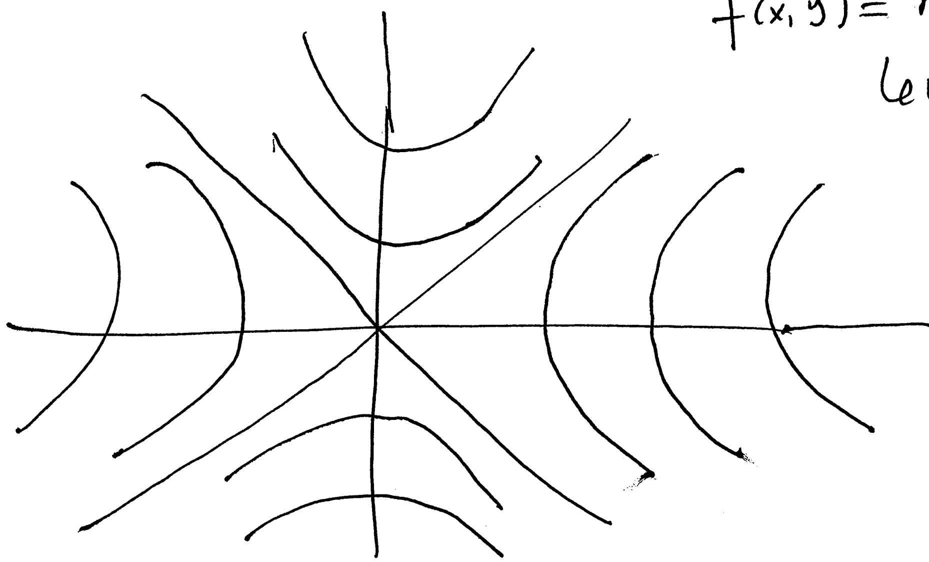
$$f(y) = \frac{2}{3} y^{1/3} \quad ; \quad \frac{\partial f}{\partial y} = -\frac{2}{9}\, y^{-2/3} \quad \text{not continuous}$$

Solution 1: $y(x) = 0$, Solution 2: $y(x) = x^{3/2}$

$y' = \frac{3}{2} x^{1/2}$; $y^{1/3} = x^{1/2}$

Isoclines: for $dy/dx = f(x,y)$ are the curves $f(x,y) = k$

level curves of $f(x,y)$



$f(x,y) = y^2 - x^2$.

Equilibrium Solution

$$\frac{dy}{dx} = f(x,y)$$

A solution of the form $y = y_0$

is called an equilibrium solution.

Ex: $$\frac{dy}{dx} = y^2 - 4$$

has the equilibrium solutions

$y(x) = 2$ $\quad$ $y(x) = -2$.