MA 262 2/3/2016 Lesson 10

Section 2.5 Gaussian Elimination.

$$3x_1 + 5x_2 - x_3 = 14$$

$$x_1 + 2x_2 + x_3 = 3$$

$$2x_1 + 5x_2 + 6x_3 = 2.$$

Step 1. Work Augmented matrix

$$\begin{bmatrix} 3 & 5 & -1 & 14 \\ 1 & 2 & 1 & 3 \\ 2 & 5 & 6 & 2 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 3 & 5 & -1 & 14 \\ 2 & 5 & 6 & 2 \end{bmatrix} \begin{array}{l} -3R_1 + R_2 \\ -2R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & -4 & 5 \\ 0 & 1 & 4 & -4 \end{bmatrix} \quad \begin{matrix} R_2 + R_3 \\ -R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>No solution.</u> !

$-8/3 \; -2$

A = matrix of coefficients

$$\begin{bmatrix} 3 & 5 & -1 \\ 1 & 2 & 1 \\ 2\cancel{8} & 5 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$ Rank 2.

$A^{\#}$ = Augmented Matrix has <u>rank 3</u>.

(1) If the rank of $A <$ rank of $A^{\#}$

the system is inconsistent

(2) rank of A = rank of $A^{\#} = n$

System has a unique solution.

(3) rank of A = rank of $A^{\#} < n$.

system has infinitely many solution.

Special Case $Ax = 0$

Rank A = Rank $A^{\#}$

If Rank $A = n$ only Solution $x = 0$

Rank $A < n$ infinitely many Sols

$$x_1 + x_2 + x_3 - x_4 = 4$$

$$x_1 - x_2 - x_3 - x_4 = 2$$

$$x_1 + x_2 - x_3 + x_4 = -2$$

$$x_1 - x_2 + x_3 + x_4 = -8$$

$$\begin{bmatrix} 1 & 1 & 1 & -1 & 4 \\ 1 & -1 & -1 & -1 & 2 \\ 1 & 1 & -1 & 1 & -2 \\ 1 & -1 & 1 & 1 & -8 \end{bmatrix}$$

$-R_1 + R_2$; $-R_1 + R_3$, $-R_1 + R_4$

$$\begin{bmatrix} 1 & 1 & 1 & -1 & 4 \\ 0 & -2 & -2 & 0 & -2 \\ 0 & 0 & -2 & 2 & -6 \\ 0 & -2 & 0 & 2 & -12 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 & 4 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 6 \\ 0 & 0 & 1 & -1 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 & 4 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & +1 & +1 & -5 \\ 0 & 0 & 1 & -1 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 & 4 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & -2 & 8 \end{bmatrix}$$

$$2x_1 - x_2 + 3x_3 - x_4 = 3$$

$$3x_1 + 2x_2 + x_3 - 5x_4 = -6$$

$$x_1 - 2x_2 + x_3 + x_4 = 6$$

$$\begin{bmatrix} 2 & -1 & 3 & -1 & 3 \\ 3 & 2 & 1 & -5 & -6 \\ 1 & -2 & 1 & 1 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 1 & 1 & 6 \\ 2 & -1 & 3 & -1 & 3 \\ 3 & 2 & 1 & -5 & -6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 1 & 1 & 6 \\ 0 & 3 & 1 & -3 & -9 \\ 0 & 8 & -2 & -8 & -24 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & -2 & 1 & 1 & 6 \\ 0 & 1 & 1/3 & -1 & -3 \\ 0 & 8 & -2 & -8 & -24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & 1 & 6 \\ 0 & 1 & 1/3 & -1 & -3 \\ 0 & 0 & -14/3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & 1 & 6 \\ 0 & 1 & 1/3 & -1 & -3 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 0 & 1 & 6 \\ 0 & 1 & 0 & -1 & -3 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & -3 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$x_3 = 0 \qquad x_2 - x_4 = -3$

$x_1 - x_4 = 0$

Find the values of α and β

such that the system

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + 4x_2 + 3x_3 = 1$$

$$x_1 + \alpha x_2 + x_3 = \beta$$

has no solution, only one solution,

infinitely many solution

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 4 & 3 & 1 \\ 1 & \alpha & 1 & \beta \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 0 & 1 & -3 \\ 0 & \alpha-2 & 0 & \beta-4 \end{bmatrix}$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 4 \\ 0 & \alpha-2 & 0 & \beta-4 \\ 0 & 0 & 1 & -3 \end{array}\right].$$

$\underline{\alpha \neq 2}$: only one Solution.

$\underline{\alpha = 2}$: $\beta = 4$; Infinitely many Solutions

$\alpha = 2$; $\beta \neq 4$, No Sols.