Definition of a Vector Space

~~Example~~

(1) V is a ~~set~~ non-empty set

(2) Scalars in $\mathbb{R}$ or $\mathbb{C}$ (use F)

(3) There exist two operations on V.

3.i: Addition: $v_1, v_2 \in V$; $v_1 + v_2 \in V$

3.ii Product by Scalar: $\lambda \in F$, $v \in V$

$$\lambda v \in V.$$

Properties of + and $\cdot$

(1) $\lambda(v_1 + v_2) = \lambda v_1 + \lambda v_2$ ~~(6) $v_1 v_2 = v_2 v_1$~~

(2) $\exists\, 0 \in V$ $0 + v = v$, $(v - v) = 0$.

(3) $1 \cdot v = v$

(4) $(\lambda t) v = \lambda t v$.

(5) $(\lambda + t) v = \lambda v + t v$.

Ex.: (1) Is the set of solutions to $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ (4)

$$Ax = 0 \qquad A = m \times n \text{ matrix}$$

a vector space?

(1) $Ax = 0$, $Ay = 0$, $A(x+y) = 0$

(2) $A(\lambda x) = \lambda Ax = 0$ $\qquad \lambda x \in S$

(3) $\lambda(x+y) = \lambda x + \lambda y.$

(2) $P = \{ ax^3 + bx^2 + cx + d = p(x) \,;\, a, b, c, d \in \mathbb{R} \}$

(2) $\{ (x, y) \,:\, x^2 + y^2 = 1 \}$

<u>Subspaces</u>: V a vector space

$S \subset V$ is a subspace if

S is itself a vector space with the same operation of V.

Examples: $V = \{ [a_{jk}]_{m \times n} = A \}$. 3

(1) $A = [a_{jk}]$; $B = [b_{jk}]$

$$A + B = [a_{jk} + b_{jk}]$$

$$\lambda A = [\lambda a_{jk}]$$

$$0 = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & 0_{jk} & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

(2) $I \subset \mathbb{R}$ is an interval. The set of all functions $f(x)$; $x \in I$. defined on I form a vector space.

(1) $(f+g)(x) = f(x) + g(x)$

(2) $(\lambda f)(x) = \lambda f(x)$

(3) $\lambda(f+g)(x) = \lambda f(x) + \lambda g(x)$

(4) $0 + f = f$.