MA 262 (1/13/2016) Lesson 2 1.4 and 1.5 1

Separable Equations:

$$P(y)\frac{dy}{dx} = q(x).$$

Examples:

$$y^2\frac{dy}{dx} = x^2.$$

$$\frac{d}{dx}y^3 = 3y^2\frac{dy}{dx}; \quad y^2\frac{dy}{dx} = \frac{1}{3}\frac{d}{dx}y^3$$

$$\frac{1}{3}\frac{d}{dx}y^3 = x^2; \quad \frac{1}{3}y^3(x) - \frac{1}{3}y^3(0) = \frac{1}{3}x^3$$

$$y^3(x) = y^3(0) + x^3.$$

$$\boxed{y(x) = \left[y^3(0) + x^3\right]^{1/3}}$$

$$y^2\frac{dy}{dx} = x^2; \quad y^2\,dy = x^2\,dx$$

$$\frac{1}{3}y^3(x) - \frac{1}{3}y^3(0) = \frac{1}{3}x^3$$

$$\boxed{y^3(x) = y^3(0) + x^3}$$

$$P(y)\, dy = q(x)\, dx$$

Integrate both sides:

Ex: $\dfrac{dy}{dx} = \dfrac{y^2}{1+x^2}$.

Solution 1: $y = 0$, equilibrium solution.

If $y \neq 0$

$$\frac{dy}{y^2} = \frac{dx}{1+x^2} \; ; \quad -\frac{1}{y} = \arctan x + C$$

$$-1/y = \arctan x + c$$

$$\boxed{y = \frac{-1}{c + \arctan x}}$$

Ex: $\dfrac{dy}{dx} = \dfrac{y(y+1)}{x}$; $x \neq 0$

Again: $y = 0$, $y = -1$, equilibrium solution.

$$\frac{dy}{y(y+1)} = \frac{dx}{x}$$

$$\frac{1}{y(y+1)} = \frac{A}{y} + \frac{B}{y+1} = \frac{A + Ay + By}{y(y+1)}$$ 3

$$A + (A+B)y = 1 \qquad A = 1, \; B = -1.$$

$$\frac{1}{y(1+y)} = \frac{1}{y} - \frac{1}{y+1}.$$

$$\int \frac{dy}{y(1+y)} = \ln|y| - \ln|y+1|.$$

$$\ln\left|\frac{y}{y+1}\right| = \ln|x| + c.$$

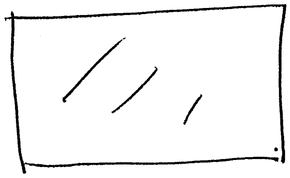
$$\left|\frac{y}{y+1}\right| = e^{c}\,|x|.$$

$$\frac{y}{y+1} = Kx. \qquad y = Kx + Kxy$$

$$\boxed{y = \frac{Kx}{1-Kx}}$$

Newton's Law of Cooling:

T_0 initial temperature



$T(t)$ = temperature of an object at time t

T_m = Room temperature.

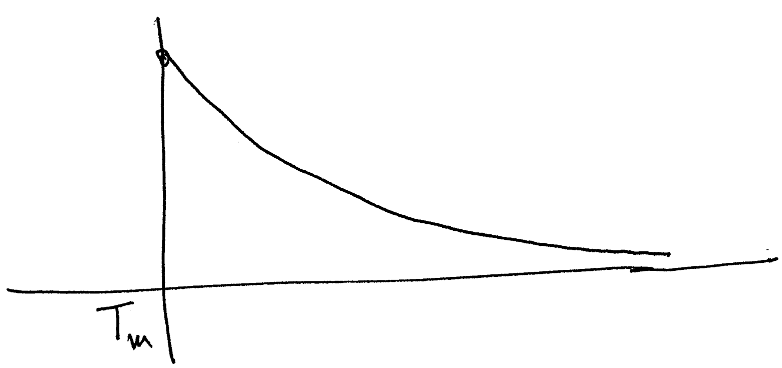
$$\frac{dT}{dt} = -k(T - T_m)$$

$$\frac{dT}{T - T_m} = -k\,dt$$

$$\lg(T - T_m) = -kt + c$$

$$T - T_m = e^c e^{-kT}$$

$$\boxed{T - T_m = (T_0 - T_m)e^{-kT}}$$



Population Growth Models

$P(t) =$ Population at time t.

$$\frac{dP}{dt} = \left[B(t) - D(t) \right] P$$

$B(t)$ - # of births

$D(t)$ - # of deaths

Malthusian Model: $B(t) - D(t) = k$.

$$\frac{dP}{dt} = kP \quad ; \quad P(0) = P_0$$

$$\boxed{P(t) = P_0 e^{kt}}$$

Logistic Model: $B(t) = r =$ Constant

$D(t) = D_0 P$

$$\frac{dP}{dt} = (r - D_0 P)\, P. \quad ; \quad P(0) = P_0$$

$r / D_0 = C$.

$$\boxed{\frac{dP}{dt} = r\left(1 - \frac{P}{C}\right) P}$$

$$\frac{dP}{(1-\frac{P}{C})P} = r\,dt$$

$$\frac{1}{(1-\frac{P}{C})P} = \frac{A}{P} + \frac{B}{1-P/C} = \frac{A - \frac{A}{C}P + BP}{P(1-P/C)}$$

$$A + (B - A/C)P = 1 \;;\quad A = 1;\quad B = A/C.$$

$$\int \frac{dP}{P\cdot(1-P/C)} = \int \frac{dP}{P} + \frac{A}{C}\int \frac{dP}{1-P/C}.$$

$$= \ln P - \ln\left|1 - P/C\right| + K.$$

$$\ln\left|\frac{P}{1-P/C}\right| + K = rt$$

$$\frac{P}{1-P/C} = e^{rt}\cdot e^{-k} \qquad \cancel{e^{-k} = A}$$

$$\frac{P}{1-P/C} = \frac{P_0}{1-P_0/C}\, e^{rt}.$$

$$A = P_0 / 1 - P_0/C$$

$$P = A\left(1 - \frac{P}{C}\right) e^{rt}$$

$$P\left(1 + \frac{A e^{rt}}{C}\right) = A e^{rt}$$

$$\boxed{P(t) = \frac{A e^{rt}}{1 + \frac{A}{C} e^{rt}}}$$