Lesson 8 : Sections 2.2 and 2.3

Systems of Linear Equations

$$A = [a_{ij}]_{m\times n} \; ; \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \; ; \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

then

$$Ax = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

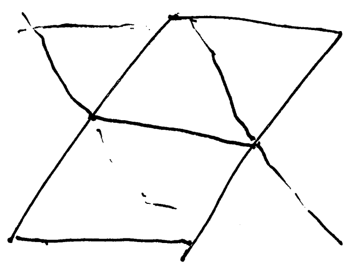
So $Ax = b$ represents a $m\times n$ system of linear equations.

A solution of this system is a vector $c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$ which satisfies

$$Ac = b.$$

$$\begin{cases} 2x_1 + 3x_2 + x_3 = 1. \\ x_1 - x_2 + x_3 = 2 \end{cases}$$

These equations represent ~~the~~ two planes in space.



Infinitely many solutions.

$$\boxed{\cancel{x_1 + 4x_2 = +1}}$$

__Def:__ A system that has at least one solution is consistent, whereas a system that has no sol is inconsistent

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Solve

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ 2x_1 + x_2 - x_3 = 2 \\ 3x_1 + 2x_2 - x_3 = 0 \end{cases}$$

Example.

$$\begin{cases} 2x_1 + 3x_2 = 1 \\ x_1 + x_2 = 3 \end{cases}$$

$x_1 = 3 - x_2$; $\quad 2(3 - x_2) + 3x_2 = 1$

$6 + x_2 = 1, \quad x_2 = -5$

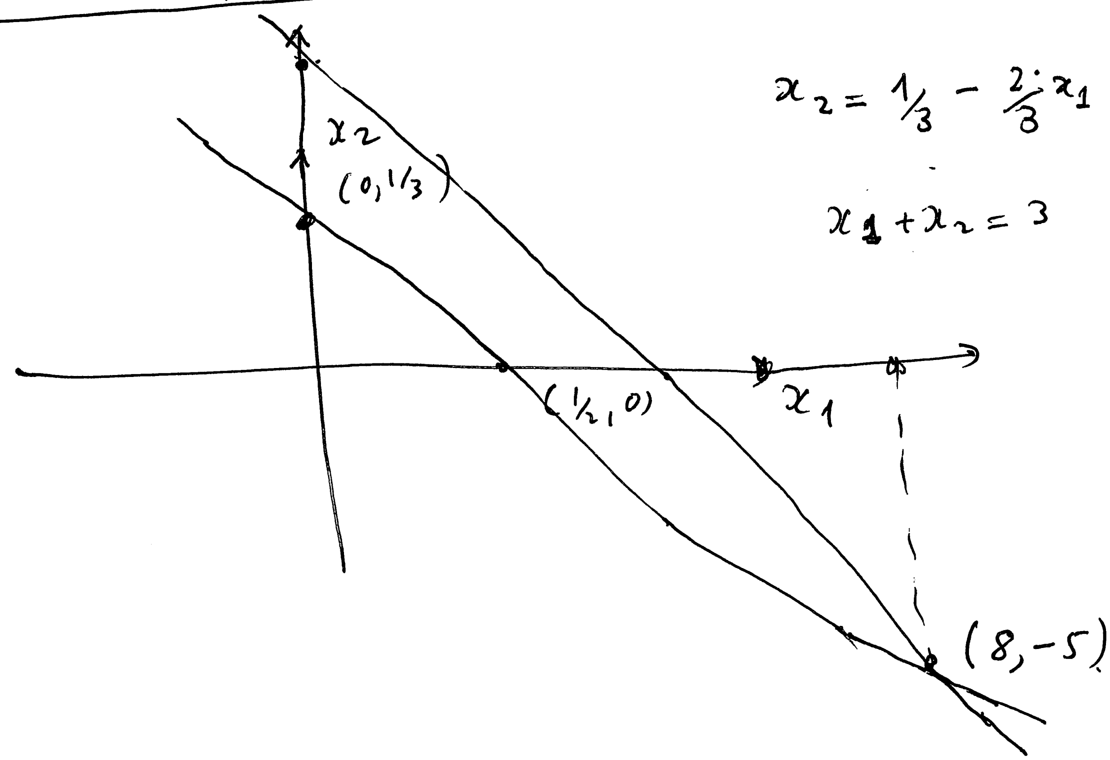
$x_1 = 8$

Solution $\quad c = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$ the order is of course important.

Geometric Interpretation.

$x_2 = 1/3 - \frac{2}{3} x_1$

$x_1 + x_2 = 3$



$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & -1 \end{bmatrix} = \text{Matrix of coefficients}$$

$$A^{\#} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 2 \\ 3 & 2 & -1 & 0 \end{bmatrix} \text{ augmented matrix.}$$

Examples.. $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$; $B = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} ; \quad BA = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$$

$$AB \neq BA.$$

Find

$$A = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

Such that $\quad A^2 + \begin{bmatrix} 0 & -3 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$

Identity matrix $\quad I_n = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots \; 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2a & 2b + ac \\ 0 & 1 & 2c \\ 0 & 0 & 1 \end{bmatrix} =$$

$$A^2 + \begin{bmatrix} 0 & -3 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & d \end{bmatrix} = \begin{bmatrix} 1 & 2a-3 & 2b+ac \\ 0 & 1 & 2c-2 \\ 0 & 0 & d \end{bmatrix}$$

$2a - 3 = 0, \quad a = 3/2; \quad 2c - 2 = 0, \quad c = 1$

$2b + ac = 0; \quad 2b + 3/2 = 1$