

Lesson 9, Section 2.4Elementary Row Operations

Solve the system:

$$x_1 + 2x_2 + x_3 = 1$$

$$x_1 - x_2 + 2x_3 = 3$$

$$2x_1 + x_2 + x_3 = 2$$

The system remains the same if we do one of the following operations

- 1.) Permute equations.
- 2.) Multiply an equation by  $c \neq 0$
- 3.) Add one equation to another one.

Instead of ~~solving~~ working with the equations we work with the augmented ~~sys~~' matrix

$$\left[ \begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 1 & -1 & 2 & 3 \\ 2 & 1 & 1 & 2 \end{array} \right]$$

multiply 1st ~~equation~~ <sup>Row</sup> by  $-1$  add to 2nd (2)

$-2$  add to 3rd.

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & 2 \\ 0 & -3 & -1 & 0 \end{bmatrix}$$

multiply 2nd row by  $-1$  add to 3rd.

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

Divide 3rd row by  $-2$ , 2nd row by  $-3$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

3rd row:  $x_3 = 1$ .

2nd row  $x_2 - \frac{1}{3}x_3 = -\frac{2}{3}$

$$\boxed{x_2 = -\frac{1}{3}}$$

$$x_1 + 2x_2 + x_3 = 1$$

(3)

$$x_1 - \frac{2}{3} + 1 = 1 \quad ; \quad \boxed{x_1 = \frac{2}{3}}$$

## Row - Echelon Matrices

1. If there are any rows consisting entirely of zeros, they are grouped together at the bottom.
2. The 1st non-zero element of any non-zero row is called a leading 1.
3. The leading 1 of any row below the 1st row is to the right of the leading 1 of the row above it.
4. The number of non-zero rows of any ~~matrix~~ row-echelon form of a matrix  $A$  is called the rank of  $A$ ,  $\text{rank}(A)$ .

## Reduced Row - Echelon

5. Any column that contains a leading 1 has zeros everywhere else.

Reduce the matrix

(4)

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 1 & -2 & 1 & 3 \\ 1 & -5 & 0 & 5 \end{bmatrix}$$

to row-echelon and reduced row-echelon

Permute rows 1 and 2.

$$\begin{bmatrix} 1 & -2 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 1 & -5 & 0 & 5 \end{bmatrix}$$

multiply  $R_1$  by  $-2$  add to 2nd row  
 $R_1$  by  $-1$  add to 3rd row

$$\begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 3 & 1 & -2 \\ 0 & -3 & -1 & 2 \end{bmatrix}$$

multiply  $R_2$  by 1 add to 3rd.

$$\begin{bmatrix} 1 & -1 & -1 & 2 \\ 3 & -2 & 0 & 7 \\ 2 & -1 & 2 & 4 \\ 4 & -2 & 3 & 8 \end{bmatrix} \begin{array}{l} -3R_1 + R_2, \\ -2R_1 + R_3 \\ -4R_1 + R_4 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 2 & 7 & 0 \end{bmatrix} \begin{array}{l} -R_2 + R_3 \\ -2R_2 + R_4 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} -R_3 + R_4$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \text{Row-echelon} \\ \text{but} \\ \text{not} \\ \text{Reduced} \\ \text{Row-echelon} \end{array}$$

$\cdot R_2 + R_1$

7

$$\begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-3R_3 + R_2 ; -4R_3 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced  
Row - echelon

~~$x_3 \in \mathbb{R}, x_2 = 5, x_1 = 7$~~