

**SEISMIC IMAGING WITH THE GENERALIZED RADON
TRANSFORM AND DOUBLE BEAMFORMING: A CURVELET
TRANSFORM PERSPECTIVE**

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A key challenge in seismic imaging reflectors from surface reflection data is sub-surface illumination, given available data coverage on the one hand and complexity of the background model (of wavespeeds) on the other hand. The imaging is, here, described by the generalized Radon transform. To address the illumination challenge we develop a method for partial reconstruction. We make use of the curvelet transform, the associated matrix representation of the generalized Radon transform, which needs to be extended in the presence of caustics, and its structure, and phase-linearization. We pair an image target with partial waveform reflection data, and develop a way to solve the matrix normal equations that connect their curvelet coefficients via diagonal approximation. Moreover, we develop an approximation, reminiscent of Gaussian beams, for the computation of the generalized Radon transform matrix elements only making use of multiplications and convolutions, given the underlying ray geometry. Throughout, we exploit the (wavenumber) multi-scale features of the dyadic parabolic decomposition underlying the curvelet transform and establish approximations that are accurate for sufficiently fine scales. The analysis we develop here has its roots in (double) beamforming and beam-stack imaging, parsimonious pre-stack Kirchhoff migration, pre-stack plane-wave (Kirchhoff) migration, and delayed-shot pre-stack migration.