

Discrete 3D Smith curvelet transform

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The theory of second microlocalization, or dyadic parabolic decomposition, has been an important development in harmonic analysis. Over the recent years, it has led to the development of new multi-scale computational techniques in image processing, constructing solutions to (hyperbolic) partial differential equations and inverse problems.

At the basis of all these developments is the frame of curvelets introduced by H. Smith and the associated class of transforms. The general concept has its roots in the theory of coherent wave packets and Fourier integral operators (Córdoba and Fefferman), and parabolic cutoffs dating back to Boutet de Monvel. Furthermore, the curvelet transform can be related to the Fourier-Bros-Iagolnitzer (FBI) transform.

We revisit Smith's frame of curvelets and discretize it making use of the USFFT as developed by Dutt and Roklin, and Beylkin. In the discretization, we directly approximate the underlying dyadic parabolic decomposition, its (rotational) symmetry, and approximate all the necessary decay estimates in phase space with arbitrary accuracy. Numerically, our transform is unitary. Moreover, if we apply the inverse transform after the forward transform, we approximate the identity matrix, as it should, and if we apply the forward transform after the inverse transform we recover the necessary decay estimates (of the matrix representing the identity operator). Another relevant aspect of our discretization is the appearance of parameters that control the tiling of phase space corresponding with the dyadic parabolic decomposition, preserving the relative parabolic scaling, while adapting to the physical problem at hand. We consider applications in exploration seismology and global seismology. For these, we need transforms in higher dimensions, that is, in dimension $n = 3, 4, 5$, while the data sets and images are inherently very large. We propose a parallel algorithm suited for such large-scale problems in the realm of high performance computing.

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