# K-THEORY OF $\Psi$DOS WITH PERIODIC SYMBOLS ON A CYLINDER

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Let $\mathfrak{A}$ denote the smallest norm-closed adjoint-invariant sub-algebra of the algebra of all bounded operators on $L^2(S^1 \times \mathbb{R})$ containing: $(i)$ all operators of multiplication by functions that extend continuously to the compactification $S^1 \times [-\infty, +\infty]$ and by continuous functions which are $2\pi$-periodic on the second variable, $(ii)$ $\Lambda = (1-\Delta)^{-1/2}$, where $\Delta$ denotes the laplacian on $S^1 \times \mathbb{R}$, $(iii)$ $\partial_\theta \Lambda$ and $(iv)$ $\partial_t \Lambda$, where $\partial_\theta$ and $\partial_t$ denote the canonical first-order derivatives on $S^1$ and on $\mathbb{R}$, respectively. This algebra contains the classical zero-order pseudodifferential operators on $S^1 \times \mathbb{R}$ with $2\pi$-periodic symbols.

I plan to start this talk with a review of some 1990 results (including a Fredholm criterion) about the structure of $\mathfrak{A}$, and to state basic definitions and theorems on C$^*$-algebra K-theory. Then I shall report on the computation, done jointly with Patrícia Hess, of the K-groups of the quotient $\mathfrak{A}/\mathfrak{K}$, with $\mathfrak{K}$ denoting the ideal of compact operators. Using an Atiyah-Singer-Fedosov index formula for the Fredholm-index of elliptic operators on a compact manifold, we managed to compute the K-theory-index associated to the principal symbol of $\mathfrak{A}$. We could not, however, directly compute the second connecting mapping for the principal symbol. We bypassed this obstacle by studying several other exact sequences (including a Pimsner-Voiculescu one) associated to certain sub-algebras of $\mathfrak{A}$. In the end, we were able to completely compute the K-groups of $\mathfrak{A}/\mathfrak{K}$.