Minicourse in Recife

Title

Spectral methods in superconductivity

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In these lectures, we would like to show how the analysis of the bottom of the spectrum of the Neumann realization of a Schrödinger operator with magnetic potential plays an important role in the understanding of the mechanism of the onset of superconductivity for a sample submitted to an external magnetic field.

The course will start with a linear part : analysis of the Schrödinger operator with magnetic field (self-adjointness, diamagnetism, asymptotic of the groundstate energy in the semi-classical regime). We will in particular analyze explicitely the spectrum for basic models with constant magnetic field in R^2, R^2_+ or the disk.

It will continue with a non-linear part and more specifically with the introduction of the Ginzburg-Landau functional (existence of minimizers, Ginzburg-Landau equation). This functional depends on two parameters κ (a characteristics of the sample) and H the intensity of the external magnetic field.

We will then discuss in function of (κ, H) the properties of the minimizers. We will focus on the case κ large and on the discussion whether the minimizer is "normal".

We will show at the end how this question can be solved through the spectral analysis presented in the first part.

If time permits, we can show applications to Liquid crystals theory or present numerical evidence of tunneling effect for problems in regular polygons.

Prerequisits. Some background in spectral theory could be useful. The standard reference is Reed-Simon. One can also find a course in Spectral theory available on my webpage http://www.math.u-psud.fr/ helffer.

The main reference is a book in preparation with S. Fournais.