Geometric Scattering Theory with Applications to Conformal and CR Geometry

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These lectures will be an exposition of recent work on the relation between conformal invariants of a compact manifold M and the scattering operator on real conformally compact manifold X, and its complex analogue, the CR-invariants on a compact CR manifold M and the scattering operator on certain complex manifolds X. In geometric scattering theory, these compact manifolds M are the boundary at infinity of a manifold X with a metric that degenerates on the boundary. The scattering operator $S(s)$ on X, for real values of the spectral parameter, is a unitary operator on $L^2(M)$. In the real case, Graham and Zworski [3] identified certain conformally invariant operators on M with the residues of the poles of the scattering matrix at certain values of the spectral parameter. A parallel construction is possible for CR-invariant operators on a CR manifold. In addition, the Q-curvature of M is obtained directly from the scattering matrix.

A brief outline of the lectures:

(1) *Lecture 1: Review of Geometric Scattering Theory: General Case.* This will be based on parts of R. Melrose's book [8]. I will discuss the idea of compact manifolds with boundary and various scattering metrics.
(2) *Lecture 2: Geometric Scattering Theory: Models I. Real Asymptotically Hyperbolic Manifolds.* Basic models of hyperbolic space, followed by a description of infinite-volume, geometrically finite hyperbolic manifolds [5], and asymptotically hyperbolic manifolds.
(3) *Lecture 3: Geometric Scattering Theory: Models I. Complex Hyperbolic Manifolds.* A discussion of the complex ball in $\mathbb{C}^m$ and its geometry and a general complex manifold with CR boundary.
(4) *Lecture 4: Conformal Manifolds and Conformal Invariants.* Description of the S-matrix and its poles following [3]. Connection with the conformal invariants described in [2].
(5) *Lecture 5: CR-Manifolds and CR-invariants.* Description of the S-matrix following [9], [4] and [6, 7]. Construction of the CR-invariant operators [1, 2] and relation to the poles of the scattering matrix.

References

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