

SHARP BOUNDS ON THE NUMBER OF THE RESONANCES AND APPLICATIONS TO THE SCATTERING THEORY

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In this mini-course, the presenter will discuss upper bounds on the number of the resonances (known also as scattering poles) for compactly supported perturbations, G , of the Euclidean Laplacian following the papers [2], [3], [5]. Recall that the resonances are the poles of the meromorphic continuation of the cut-off resolvent

$$R_\chi(\lambda) = \chi(G - \lambda^2)^{-1}\chi$$

from $\{\text{Im } \lambda < 0\}$ to the whole complex plane \mathbf{C} if the dimension n is odd, and to the Riemann surface, \mathbf{C}^* , of the logarithm if n is even. The resonances form a discrete set, \mathcal{R} , of points which may accumulate only at infinity, and the multiplicity of a resonance λ_j is given by

$$\text{rank} \int_{|\lambda - \lambda_j| = \epsilon} \lambda R_\chi(\lambda) d\lambda < +\infty.$$

The following counting functions play an important role in the scattering theory:

$$N(r) = \#\{\lambda_j \in \mathcal{R} \subset \mathbf{C} : |\lambda_j| \leq r\}, \quad r \geq 1,$$

if n is odd, and

$$N(r, a) = \#\{\lambda_j \in \mathcal{R} \subset \mathbf{C}^* : |\lambda_j| \leq r, |\arg \lambda_j| \leq a\}, \quad r, a \geq 1,$$

if n is even, where the resonances are counted with the multiplicity. In this mini-course we will give a simple proof of the following upper bounds:

$$N(r) \leq Cr^n,$$

$$N(r, a) \leq Ca(r^n + (\log a)^n).$$

As a consequence of these bounds, the lecturer will prove the Weyl asymptotic for the scattering phase in the obstacle scattering following [1]. Estimates of the resolvent will be also discussed. We will also reveal the connection between those estimates and the local energy decay of the solution of the corresponding wave equation. We will also review estimates on the asymptotic distribution of resonances in various domains in the complex plane.

REFERENCES

- [1] R. MELROSE, *Weyl asymptotic for the phase in obstacle scattering*, Commun. PDE **13** (1988), 1431-1439.
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- [4] G. VODEV, *Resonances in the Euclidean scattering*, Cubo Mat. Educacional **3** (2001), 317-360.
- [5] M. ZWORSKI, *Sharp polynomial bounds on the number of scattering poles*, Duke Math. J. **59** (1989), 311-323.