Harmonic experiments to model fracture induced anisotropy

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work in collaboration with J. M. Carcione and S. Picotti, (OGS), Trieste, Italy.
Fractures are common in the earth’s crust due to different factors, for instance, tectonic stresses and natural or artificial hydraulic fracturing caused by a pressurized fluid.

Seismic wave propagation through fractures and cracks is an important subject in exploration and production geophysics, earthquake seismology and mining.

Fractures constitute the sources of earthquakes, and hydrocarbon and geothermal reservoirs are mainly composed of fractured rocks.
Modeling fractures requires a suitable interface model. Schoenberg (JASA (1980), GP (1983)) proposed the so-called linear-slip boundary condition model (LSBC), based on the discontinuity of the displacement across the fractures. (Schoenberg’s model).

A generalization of the (LSBC) (Carcione, JGR (1996)) states that across a fracture stress components are proportional to the displacement and velocity discontinuities through the specific stiffnesses and specific (viscosities, respectively.)
Displacement discontinuities conserve energy, while velocity discontinuities generate energy loss at the fractures. The specific viscosity accounts for the presence of a liquid under saturated conditions, introducing a viscous coupling between both sides of a fracture.

Schoemberg’s theory predicts that a dense set of parallel plane fractures behaves as a Transversely Isotropic Viscoelastic (TIV) medium if the dominant wavelength of the traveling waves is much larger than the distance between the fractures.
Schoenberg’s model has never been simulated with a numerical method.

To test the theory, in the context of Numerical rock physics we developed a novel numerical solver that can be used in more general situations.

Numerical rock physics offer an alternative to laboratory measurements.

Numerical experiments are inexpensive, repeatable, essentially free from experimental errors and can easily be run using alternative models of the materials being analyzed.
To determine the complex stiffness coefficients of the equivalent TIV medium, we solve a set of boundary value problems (BVP’s) for the wave equation of motion in the frequency-domain using the finite-element method (FEM).

The BVP’s represent harmonic tests at a finite number of frequencies on a sample having a dense set of fractures, modeled using the LSBC.
The equivalent TIV medium. I

Consider a viscoelastic isotropic background medium having a set of parallel (horizontal) fractures and its description in the space-frequency domain.

\( u, e_{ij}(u), \sigma_{ij}(u) \): frequency domain displacement vector, strain components and stress components of the background medium.

The stress-strain relations and equations of motion:

\[
\sigma_{jk}(u) = \lambda \delta_{jk} \nabla \cdot u + 2 \mu e_{jk}(u)
\]

\[
\rho \omega^2 u(x, z, \omega) + \nabla \cdot \sigma[u(x, z, \omega)] = 0
\]

\( \delta_{jk} \): Kroenecker delta  
\( \lambda, \mu \): complex Lamé constants  
\( \rho \): mass density.
$x_1$ and $x_3$: horizontal and vertical coordinates, respectively.

When a dense set of parallel fractures is present, the medium behaves as a TIV medium with $x_3$-axis of symmetry at long wavelengths.

$\tau_{ij}, \epsilon_{ij}$: stress and strain tensors of the equivalent TIV medium

**Stress-strain relations:**

\begin{align*}
\tau_{11}(u) &= p_{11} \epsilon_{11}(u) + p_{12} \epsilon_{22}(u) + p_{13} \epsilon_{33}(u), \\
\tau_{22}(u) &= p_{12} \epsilon_{11}(u) + p_{11} \epsilon_{22}(u) + p_{13} \epsilon_{33}(u), \\
\tau_{33}(u) &= p_{13} \epsilon_{11}(u) + p_{13} \epsilon_{22}(u) + p_{33} \epsilon_{33}(u), \\
\tau_{23}(u) &= 2p_{55} \epsilon_{23}(u), \\
\tau_{13}(u) &= 2p_{55} \epsilon_{13}(u), \quad \tau_{12}(u) = 2p_{66} \epsilon_{12}(u).
\end{align*}
Schoenberg’s theory predicts that if the background medium is homogeneous, the stiffnesses $p_{1ij}$’s are given by

\begin{align}
    p_{11} &= p_{22} = E - \lambda^2 Z_N c_N, \\
    p_{12} &= \lambda - \lambda^2 Z_N c_N, \\
    p_{13} &= \lambda c_N, \\
    p_{33} &= E c_N, \\
    p_{55} &= \mu c_T, \\
    p_{66} &= \mu.
\end{align}

where

\begin{align}
    c_N &= (1 + EZ_N)^{-1}, \\
    c_T &= (1 + \mu Z_T)^{-1},
\end{align}

$Z_N$ and $Z_T$: normal and tangential complex compliances of the fractures

$E = \lambda + 2\mu$. 
The theory assumes that the distance between fractures is much smaller than the wavelength of the signal and that the boundary condition is the same for all the fractures. Moreover, the theory assumes that the fracture distance is constant.

The numerical solver may consider an inhomogeneous background medium, unequal fracture distances and dissimilar boundary conditions at the fractures surfaces.

The $p_{ij}$'s are the complex and frequency-dependent stiffnesses to be determined numerically with the harmonic experiments.
\( \Omega = (0, D)^2 \): a square sample of boundary \( \Gamma = \Gamma^L \cup \Gamma^R \cup \Gamma^B \cup \Gamma^T \), where

\[
\begin{align*}
\Gamma^L &= \{ (x, z) \in \Gamma : x = 0 \}, \\
\Gamma^R &= \{ (x, z) \in \Gamma : x = D \}, \\
\Gamma^B &= \{ (x, z) \in \Gamma : z = 0 \}, \\
\Gamma^T &= \{ (x, z) \in \Gamma : z = D \}.
\end{align*}
\]

\( \Gamma^{(f,l)}, l = 1, \ldots, J^{(f)} \): a set of \( J^{(f)} \) horizontal fractures each one of length \( D \) in our domain \( \Omega \).

This set of fractures divides \( \Omega \) in a set of nonoverlapping rectangles \( R^{(l)}, l = 1, \ldots, J^f + 1 \), so that

\[
\Omega = \bigcup_{l=1}^{J^{(f)}+1} R^{(l)}.
\]
Consider a fracture $\Gamma^{(f,l)}$ and the two rectangles $R^{(l)}$ and $R^{(l+1)}$ having as a common side $\Gamma^{(f,l)}$.

$n_{l,l+1}, m_{l,l+1}$: the unit outer normal and a unit tangent (oriented counterclockwise) on $\Gamma^{(f,l)}$ from $R^{(l)}$ to $R^{(l+1)}$, such that $\{n_{l,l+1}, m_{l,l+1}\}$ is an orthonormal system on $\Gamma^{(f,l)}$.

Set $u^{(l)} = u|_{R^{(l)}}$: restriction of $u$ to $R^{(l)}$, and let

$$[u] = (u^{(l)} - u^{(l+1)}) |_{\Gamma^{(f,l)}}$$

denote the jump of $u$ at $\Gamma^{(f,l)}$. 
The boundary conditions (B.C.) at the fractures $\Gamma^{(f,l)}$ are the stress continuity and the condition that stress components be proportional to the displacement and velocity discontinuities through specific stiffnesses and viscosities, respectively. Thus,

$$\sigma(\mathbf{u}^{(l)}) \mathbf{n}_{l,l+1} = \sigma(\mathbf{u}^{(l+1)}) \mathbf{n}_{l,l+1} \Gamma^{(f,l)},$$

$$(\sigma(\mathbf{u}^{(l)}) \mathbf{n}_{l,l+1} \cdot \mathbf{n}_{l,l+1}, \sigma(\mathbf{u}^{(l)}) \mathbf{n}_{l,l+1} \cdot \mathbf{m}_{l,l+1})^T$$

$$= \mathbf{S}^{(l)}(\omega) (\mathbf{[u]} \cdot \mathbf{n}_{l,l+1}, \mathbf{[u]} \cdot \mathbf{m}_{l,l+1})^T \Gamma^{(f,l)}, \ l = 1, \cdots, J^{(f)}.$$ 

where $^T$ indicates the transpose.
If $L$ denotes the average distance between the fractures, the matrix $S^{(l)}(\omega)$ has the form

$$S^{(l)}(\omega) = \begin{pmatrix} (LZ_N^l)^{-1} & 0 \\ 0 & (LZ_T^{(l)})^{-1} \end{pmatrix}$$

(9)

The compliances $Z$ ($Z_N$ or $Z_T$) are complex and frequency-dependent and can be expressed as

$$Z^{-1} = L(\kappa + i\omega\eta),$$

where $\kappa$ is a specific stiffness and $\eta$ is a specific viscosity, having dimensions of stiffness and viscosity per unit length, respectively.
For $p_{33}$ we solve the wave equation in $\Omega$ using the fracture B. C.'s with the additional B. C.'s

$$\sigma(u) n \cdot n = -\Delta P, \quad \Gamma^T, \quad (10)$$

$$\sigma(u) n \cdot m = 0, \quad \Gamma, \quad (11)$$

$$u \cdot \nu = 0, \quad \Gamma^L \cup \Gamma^R \cup \Gamma^B. \quad (12)$$

In this experiment $\epsilon_{11}(u) = \epsilon_{22}(u) = 0.$
Denoting by $V$ the original volume of the sample, its (complex) oscillatory volume change, $\Delta V(\omega)$, we note that in the quasistatic case

$$\frac{\Delta V(\omega)}{V} = -\frac{\Delta P}{p_{33}(\omega)},$$

The computed average vertical displacement $u_{3,T}^s(\omega)$ suffered by the boundary $\Gamma^T$ allows us to use the estimate

$$\Delta V(\omega) \approx D u_{3,T}^s(\omega),$$

from where we can determine $p_{33}(\omega)$. 
For $p_{11}$ we solve the wave equation with the fracture B. C.'s and the additional B. C.'s

\[
\sigma(u) \cdot n \cdot n = -\Delta P, \quad \Gamma^R,
\]
\[
\sigma(u) \cdot n \cdot m = 0, \quad \Gamma,
\]
\[
u \cdot n = 0, \quad \Gamma^L \cup \Gamma^B \cup \Gamma^T.
\]

In this experiment, $\varepsilon_{33} = \varepsilon_{22} = 0$ and this experiment determines $p_{11}$ computing the volume change in the same way indicated for $p_{33}$. 
For $\rho_{55}$ we solve the wave equation with the fracture B. C.'s and the additional B. C.'s

$$\sigma \cdot m = g, \quad \Gamma^T \cup \Gamma^L \cup \Gamma^R,$$

$$u = 0, \quad \Gamma^B,$$

where

$$g = \begin{cases} 
(0, \Delta G), & \Gamma^L, \\
(0, -\Delta G), & \Gamma^R, \\
(\Delta G, 0), & \Gamma^T.
\end{cases}$$
Let $\theta(\omega)$: angle between the original positions of the lateral boundaries and the location after applying the shear stresses.

To estimate $\theta(\omega)$, we compute the average horizontal displacement $u^T_1(\omega)$ at the boundary $\Gamma^T$ and use that

$$\tan[\theta(\omega)] \approx \frac{u^T_1(\omega)}{D}.$$ 

Thus, the change in shape of the rock sample allow us to determine $p_{55}(\omega)$ from the relation (Kolsky, 1963)

$$\tan[\theta(\omega)] = \frac{\Delta G}{p_{55}(\omega)}.$$
For \( p_{13} \) we solve the wave equation with the fracture B. C.’s and the additional B. C.’s

\[
\sigma(u) \cdot n \cdot n = -\Delta P, \quad \Gamma^R \cup \Gamma^T,
\]
\[
\sigma(u) \cdot n \cdot m = 0, \quad \Gamma,
\]
\[
u \cdot n = 0, \quad \Gamma^L \cup \Gamma^B.
\]

In this experiment \( \epsilon_{22} = 0 \), and from the stress-strain relations at the macroscale we get
\[ \tau_{11} = p_{11}\epsilon_{11} + p_{13}\epsilon_{33}, \]
\[ \tau_{33} = p_{13}\epsilon_{11} + p_{33}\epsilon_{33}, \]

\( \epsilon_{11}, \epsilon_{33} \): macroscopic strain components at the right lateral side and top side of the sample, respectively.

Then using that \( \tau_{11} = \tau_{33} = -\Delta P \) we obtain

\[ p_{13}(\omega) = \frac{p_{11}\epsilon_{11} - p_{33}\epsilon_{33}}{\epsilon_{11} - \epsilon_{33}}. \]
A variational formulation

Test space for $p_{33}(\omega)$:

$$W_{33}(\Omega) = \{ v \in [L^2(\Omega)]^2 : v|_{R(l)} \in [H^1(R^{(l)})]^2, \ v \cdot n = 0 \ \text{on} \ \Gamma^L \cup \Gamma^R \cup \Gamma^B \},$$

To determine $p_{33}(\omega)$: find $u^{(33)} \in W_{33}(\Omega)$ such that:

$$-\omega^2(u, v) + \sum_{l=1}^{J^{(f)}+1} \sum_{s,t=1,3} (\sigma_{st}(u), \epsilon_{st}(v))_{R(l)}$$

$$+ \sum_{l=1}^{J^{(f)}} \left[ \left\langle [LZ^{(l)}]^{-1}[u]_3, [v]_3 \right\rangle_{\Gamma(f,l)} + \left\langle [LZ_T^{(l)}]^{-1}[u]_1, [v]_1 \right\rangle_{\Gamma(f,l)} \right]$$

$$= - \langle \Delta P, v \cdot n \rangle_{\Gamma_T}, \quad \forall v \in W_{33}(\Omega). \quad (14)$$

Similar formulations hold for the other $p_{IJ}$'s
The FE variational formulation uses **bilinear elements** to compute approximate solution of the BVP, and is explained in Santos et al. (CMAME, 2012, submitted), where apriori error estimates which are optimal for the regularity of the solution are given.

Results on the existence and uniqueness of the continuous and discrete BVP’s are also given in that reference.

The error is of the order of $h$ in the $L^2$-norm and of the order of $h^{1/2}$ both in the interior energy norm and in the $L^2$-norm on the set of fractures, where $h$ is the diameter of the elements.
Numerical experiments.

We consider the data provided by the laboratory experiments of Chichinina et al. (TPM, 2009). The background medium is isotropic with \( \lambda = 10 \text{ GPa}, \mu = 3.9 \text{ GPa} \) and \( \rho = 2300 \text{ kg/m}^3 \).

The simulations to determine the \( p_{ij} \)'s have fracture distance \( L = 1 \text{ cm} \), grid spacing \( h = 0.5 \text{ cm} \) and a frequency \( f_0 = 25 \text{ Hz} \).

\( p_{11}, p_{13} \) and \( p_{33} \): 29 equally spaced fractures, \( 60 \times 60 \) mesh.

\( p_{55} \): 14 fractures, \( 30 \times 30 \) mesh.

Experimental values of \( Z_N \) and \( Z_T \) for wet fractures scaled to seismic frequencies:

\[
Z_{N}^{-1} = [34 + i(f/f_0) \times 24.7] \text{ GPa} \quad Z_{T}^{-1} = [15.5 + i(f/f_0) \times 11.3] \text{ GPa}
\]

Determination of reliable fracture parameters needs measurements at the seismic range (experimental data was obtained at 100 kHz).
Validation of the FE method. Phase velocities as function of frequency for wet fractures.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Phase velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Numerical</strong></td>
</tr>
<tr>
<td>50</td>
<td>2200</td>
</tr>
<tr>
<td>100</td>
<td>2300</td>
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<tr>
<td>150</td>
<td>2400</td>
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<td>200</td>
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<td>250</td>
<td>2600</td>
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<tr>
<td>300</td>
<td>2700</td>
</tr>
</tbody>
</table>

“11” and “33” refer to the qP wave along and perpendicular to the fracture plane. “55” refers to the qS wave perpendicular to the fracture plane. A very good fit is observed. qP waves along the fracture plane (“11”) travel faster than qP waves travelling perpendicular to the fractures (“33”).
“11” and “33” refer to the qP wave along and perpendicular to the fracture plane. “55” and “66” refer to the qS and SH waves perpendicular and along the fracture plane. The SH wave is lossless. A very good fit is observed. qP waves along the fracture plane (“11”) suffer lower attenuation than qP waves travelling perpendicular to the fractures (“33”).
Fractures at varying pore fluid pressure.

Daley et al. (GPY, 2006) suggest to take high values of fracture compliance at low normal effective stress $\sigma = pc - pp$, where $pc$ is the confining pressure and $pp$ the pore pressure.

For a constant $pc = 30$ MPa, we consider two pore pressures 5 MPa and 25 MPa, normal and overpressure values, respectively. Using their model, we obtain, at 25 Hz,

\begin{align*}
pp = 5 \text{ MPa}, & \quad Z_N^{-1} = (23.1 + 5.9i) \text{ GPa}, \quad Z_T^{-1} = (75 + 9.4i) \text{ GPa}, \\
pp = 28 \text{ MPa}, & \quad Z_N^{-1} = (14.4 + 3.6i) \text{ GPa}, \quad Z_T^{-1} = (21 + 2.6i) \text{ GPa},
\end{align*}

We consider a set of equispaced fractures with $L = 1$ cm and 80 % binary fractal variations of $Z_N$ and $Z_T$ around these mean values.
Real part of fractal $Z_N^{-1}$ at pore pressure 28 MPa.

80 \% binary fractal variations of $Z_N$ around the mean value 23.1 GPa
Phase velocity as function of frequency for fractal ZN ZT, wet fractures. Confining pressure is 30 MPa. Pore pressures are 5 MPa and 28 MPa.

\[ \sigma = pc - pp \] is the effective normal stress, \( pc \) = confining pressure, \( pp \) = pore pressure. “11” and “33” refer to the qP wave along and perpendicular to the fracture plane. Higher pore pressure (circles) implies lower phase velocity. The “33” qP wave is the one more affected by overpressure.
σ = pc − pp is the effective normal stress, pc = confining pressure, pp = pore pressure. “11” and “33” refer to the qP wave along and perpendicular to the fracture plane. Attenuation is stronger in the overpressured case (circles) for “33” waves.
“11” and “33” refer to the qP wave along and perpendicular to the fracture plane. Phase velocities in the fractal case are lower than those obtained with the mean values. The “33” qP wave is the one more affected by the heterogeneities.
“11” and “33” refer to the qP wave along and perpendicular to the fracture plane. Dissipation factor of the “33” qP wave is more affected by the heterogeneities, showing lower values in the fractal case.
Next we consider 50% binary fractal variations of the background Lamé constants $\lambda$ and $\mu$ with respect to the mean values 10 GPa and 3.9 GPa, respectively.
“11” and “33” refer to the qP wave along and perpendicular to the fracture plane. Phase velocities are lower for the fractal case for both “11” and “33” qP waves. Concerning attenuation, for qP “33” waves is lower than in the uniform background case, while attenuation for qP “11” waves is not affected by the fractal background.
CONCLUSIONS.

- Schoenberg’s theory predicts that an homogeneous background containing a dense set of horizontal parallel fractures behaves like a TIV medium at long wavelengths.

- We presented a collection of novel FE harmonic experiments to test and validate the theory.

- The methodology was applied to a case where there is no analytical solution, such as fractal variations of the fracture compliances at different pore pressures and fractal Lamé parameters.

- In particular, it is shown that attenuation can be an indicator of overpressure with higher values at high pore pressures.
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THANKS FOR YOUR ATTENTION !!!!