Q-anisotropy of qP waves in finely-layered media

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ABSTRACT

Finely-layered media behaves as a transversely isotropic medium at long wavelengths. If the constituent media are anelastic, Q-anisotropy is described by Postma averaging for two periodic layers and by Backus averaging for an stationary sequence of many layers. In order to test the theory, we perform numerical simulations of wave propagation in a periodic sequence of sandstone and limestone and compute the Q-factor of qP waves as a function of the propagation direction.

INTRODUCTION

Most geological systems can be modeled as fine layering, which refers to the case where the dominant wavelength of the pulse is much larger than the thicknesses of the individual layers. When this occurs, the medium is effectively transversely isotropic. The first to obtain a solution for this problem was Bruggeman (1937). Other investigators studied the problem using different approaches, e.g., Riznichenko (1949) and Postma (1955), who considered a two-constituent periodically layered medium. Later, Backus (1962) obtained the average elasticity constants in the case where the single layers are transversely isotropic with the symmetry axis perpendicular to the layering plane. Moreover, he assumed stationarity; that is, in a given length of composite medium much smaller than the wavelength, the proportion of each material is constant (periodicity is not required). The equations were further generalized by Schoenberg and Muir (1989) for anisotropic single constituents.

Backus averaging for the lossless case has been verified numerically by Carcione et al. (1991). They found that the minimum ratio between the P-wave dominant-pulse wavelength and the layer thickness depends on the contrast between the constituents. For instance, for a periodic sequence of epoxy-glass it is around 8, and for sandstone-limestone (which has a lower reflection coefficient) it is between 5 and 6. In any case, an optimal ratio can be found for which the equivalence between a finely layered medium and a homogeneous transversely isotropic medium is valid. Carcione (1992) generalized Backus averaging to the anelastic case, obtaining the first model for Q-anisotropy (see Carcione, 2007). Analyses on sequences of sandstone-limestone and shale-limestone with different degrees of anisotropy indicate that the quality factors of the shear modes are more anisotropic than the corresponding phase velocities, cusps of the qSV mode are more pronounced for low frequencies and midrange proportions, and in general, attenuation is higher in the direction perpendicular to layering or close to it, provided that the material with lower velocity is the more dissipative. Other alternative models of Q-anisotropy were proposed by Carcione and Cavallini (1994; 1995) and Carcione et al. (1998). A brief description of all these models can be found in Carcione (2007).

In order to test Backus averaging for Q-anisotropy of qP waves, we perform numerical simulations and obtain the quality factor using the spectral-ratio and frequency-shift methods (e.g., Picotti and Carcione, 2006). The attenuation model is the Zener viscoelastic stress-strain relation, and the synthetic seismograms are obtained by a full-wave solver based on the Fourier pseudospectral method to compute the spatial derivatives (Carcione et al., 1988; Carcione, 2007).

BACKUS AVERAGING

Fine layering on a scale much finer than the dominant wavelength of the signal yields effective anisotropy, whose elasticity constants are given by Backus averaging (Backus, 1962). Carcione (1992) uses this approach and the correspondence principle (e.g., Carcione, 2007) to study the anisotropic characteristics of attenuation in viscoelastic finely layered media. Here, we consider that each medium be isotropic and anelastic with complex Lamé constants given by

$$\lambda(\omega) = \rho \left(c_P^2 - \frac{4}{3} c_S^2 \right) M_1(\omega) - \frac{2}{3} \rho c_S^2 M_2(\omega) \quad \text{and} \quad \mu(\omega) = \rho c_S^2 M_2(\omega), \tag{1}$$

where ω is the angular frequency, M_1 and M_2 are the dilatational and shear complex moduli, respectively, c_P and c_S are the elastic high-frequency limit compressional- and shear-wave velocities, and ρ is the density. (In Carcione (1992), the relaxed moduli correspond to the elastic limit.)

The dilatational modulus is

$$k = \lambda + \frac{2}{3}\mu = \rho \left(c_P^2 - \frac{4}{3} c_S^2 \right) M_1(\omega) \tag{2}$$

and the P-wave modulus is given by

$$E = k + \frac{4}{3}\mu. \tag{3}$$

According to Carcione (1992), the equivalent viscoelastic transversely isotropic medium is defined by the following complex stiffnesses:

$$\begin{aligned} p_{11} &= \langle E - \lambda^2 E^{-1} \rangle + \langle E^{-1} \rangle^{-1} \langle E^{-1} \lambda \rangle^2 \\ p_{33} &= \langle E^{-1} \rangle^{-1} \\ p_{13} &= \langle E^{-1} \rangle^{-1} \langle E^{-1} \lambda \rangle \\ p_{55} &= \langle \mu^{-1} \rangle^{-1} \\ p_{66} &= \langle \mu \rangle, \end{aligned} \tag{4}$$

where $\langle \cdot \rangle$ denotes the thickness weighted average. In the case of a periodic sequence of two alternating layers, equations (4) are similar to those of Postma (1955), who considered lossless layers.

We can use any complex moduli to describe the anelastic properties of the medium. For numerical-modeling purposes, the simplest realistic model is a single Zener element describing each anelastic deformation mode (identified by the index ν), whose (dimensionless) complex moduli can be expressed as

$$M_\nu(\omega) = \frac{\sqrt{Q_{0\nu}^2 + 1} - 1 + \mathrm{i}\omega Q_{0\nu}\tau_0}{\sqrt{Q_{0\nu}^2 + 1} + 1 + \mathrm{i}\omega Q_{0\nu}\tau_0}, \tag{5}$$

where $1/\tau_0$ is the central frequency of the relaxation peak, and $1/Q_{0\nu}$ is the maximum dissipation factor. The dilatational, S-wave and P-wave quality factors of each single isotropic layer are respectively given by

$$Q_1 = \frac{\mathrm{Re}(k)}{\mathrm{Im}(k)}, \quad Q_S = Q_2 = \frac{\mathrm{Re}(\mu)}{\mathrm{Im}(\mu)}, \quad \text{and} \quad Q_P = \frac{\mathrm{Re}(E)}{\mathrm{Im}(E)}. \tag{6}$$

We consider homogeneous viscoelastic waves (Carcione, 2007). The qP-wave complex velocity is the key quantity to obtain the phase velocity and quality factor of the equivalent anisotropic medium. It is given by

$$\begin{aligned} v &= (2\rho)^{-1/2}\sqrt{p_{11}l_1^2 + p_{33}l_3^2 + p_{55} + A} \\ A &= \sqrt{[(p_{11} - p_{55})l_1^2 + (p_{55} - p_{33})l_3^2]^2 + 4[(p_{13} + p_{55})l_1 l_3]^2}. \end{aligned} \tag{7}$$

(Auld, 1990; e.g., Carcione, 2007), where $l_1 = \sin\theta$ and $l_3 = \cos\theta$ are the directions cosines, and θ is the propagation angle between the wavenumber vector and the symmetry axis.

The phase velocity is simply

$$v_p = \left[\frac{1}{\mathrm{Re}(v)}\right]^{-1}. \tag{8}$$

The energy-velocity vector is given by

$$\frac{\mathbf{v}_e}{v_p} = (l_1 + l_3 \cot\psi)^{-1}\hat{\mathbf{e}}_1 + (l_1 \tan\psi + l_3)^{-1}\hat{\mathbf{e}}_3. \tag{9}$$

where

$$\tan\psi = \frac{\mathrm{Re}(\beta^* X + \xi^* W)}{\mathrm{Re}(\beta^* W + \xi^* Z)}, \tag{10}$$

defines the angle between the energy-velocity vector and the z-axis,

$$\begin{aligned} \beta &= \mathrm{pv}\sqrt{-B-A}, \\ \xi &= \mathrm{pv}\sqrt{B-A}, \\ B &= p_{11}l_1^2 - p_{33}l_3^2 + p_{55}\cos 2\theta \end{aligned} \tag{11}$$

and

$$\begin{aligned} W &= p_{55}(\xi l_1 + \beta l_3), \\ X &= \beta p_{11} l_1 + \xi p_{13} l_3, \\ Z &= \beta p_{13} l_1 + \xi p_{33} l_3 \end{aligned} \tag{12}$$

(Carcione, 2007). We have the property

$$v_p = v_e \cos(\psi - \theta), \tag{13}$$

where $v_e = |\mathbf{v}_e|$. The quality factor is given by

$$Q = \frac{\mathrm{Re}(v^2)}{\mathrm{Im}(v^2)}. \tag{14}$$

The following anisotropy coefficient can be defined

$$\epsilon_Q = \frac{Q_{11} - Q_{33}}{2Q_{33}}, \tag{15}$$

where

$$Q_{11} = \frac{\mathrm{Re}(p_{11})}{\mathrm{Im}(p_{11})} \quad \text{and} \quad Q_{33} = \frac{\mathrm{Re}(p_{33})}{\mathrm{Im}(p_{33})} \tag{16}$$

are the quality factors along layering and perpendicular to layering, respectively.

The polar curve given by $(\sin\psi, \cos\psi)Q$ is compared to the results of the numerical simulations.

EQUATIONS OF MOTION

The time-domain equations for propagation in a 2-D heterogeneous isotropic and viscoelastic medium can be found in Carcione (2007). The two-dimensional velocity-stress equations for anelastic propagation in the (x, z)-plane, assigning one relaxation mechanism to dilatational anelastic deformations ($\nu = 1$) and one relaxation mechanism to shear anelastic deformations ($\nu = 2$), can be expressed by

i) Euler-Newton's equations:

$$\dot{v}_x = \frac{1}{\rho}(\sigma_{xx,x} + \sigma_{xz,z}) + f_x, \tag{17}$$

$$\dot{v}_z = \frac{1}{\rho}(\sigma_{xz,x} + \sigma_{zz,z}) + f_z, \tag{18}$$

where v_x and v_z are the particle velocities, σ_{xx}, σ_{zz} and σ_{xz} are the stress components, ρ is the density and f_x and f_z are the body forces. A dot above a variable denotes time differentiation, and the subindices $,x$ and $,z$ indicate spatial derivatives with respect to the Cartesian coordinates.

ii) Constitutive equations:

$$\dot{\sigma}_{xx} = k_\infty(v_{x,x} + v_{z,z} + e_1) + \mu_\infty(v_{x,x} - v_{z,z} + e_2), \tag{19}$$

$$\dot{\sigma}_{zz} = k_\infty(v_{x,x} + v_{z,z} + e_1) - \mu_\infty(v_{x,x} - v_{z,z} + e_2), \tag{20}$$

$$\dot{\sigma}_{xz} = \mu_\infty(v_{x,z} + v_{z,x} + e_3), \tag{21}$$

where e_1, e_2 and e_3 are memory variables, and $k_\infty = \rho(c_P^2 - 4c_S^2/3)$ and $\mu_\infty = \rho c_S^2$ are the unrelaxed (high-frequency) bulk and shear moduli, respectively.

iii) Memory-variable equations:

$$\dot{e}_1 = \left(\frac{1}{\tau_\epsilon^{(1)}} - \frac{1}{\tau_\sigma^{(1)}}\right)(v_{x,x} + v_{z,z}) - \frac{e_1}{\tau_\sigma^{(1)}}, \tag{22}$$

$$\dot{e}_2 = \left(\frac{1}{\tau_\epsilon^{(2)}} - \frac{1}{\tau_\sigma^{(2)}}\right)(v_{x,x} - v_{z,z}) - \frac{e_2}{\tau_\sigma^{(2)}}, \tag{23}$$

$$\dot{e}_3 = \left(\frac{1}{\tau_\epsilon^{(2)}} - \frac{1}{\tau_\sigma^{(2)}}\right)(v_{x,z} + v_{z,x}) - \frac{e_3}{\tau_\sigma^{(2)}}, \tag{24}$$

where $\tau_\sigma^{(\nu)}$ and $\tau_\epsilon^{(\nu)}$ are material relaxation times, corresponding to dilatational ($\nu = 1$) and shear ($\nu = 2$) deformations. The relaxation times can be expressed as

$$\tau_\epsilon^{(\nu)} = \frac{\tau_0}{Q_0^{(\nu)}}\left(\sqrt{{Q_0^{(\nu)}}^2 + 1} + 1\right), \quad \tau_\sigma^{(\nu)} = \tau_\epsilon^{(\nu)} - \frac{2\tau_0}{Q_0^{(\nu)}}. \tag{25}$$

ESTIMATION OF THE QUALITY FACTOR

To estimate the quality factor from the synthetic time histories we use two methods (e.g., Picotti and Carcione, 2006): the classical spectral-ratio method and the frequency-shift method. Let $S(\omega)$ and $R(\omega)$ be the amplitude spectra at the source and at the receiver, respectively, which are separated by a distance d. The spectral-ratio method is based on the fact that, if the medium is homogeneous and the geometrical spreading factor G is frequency independent, the relation between the logarithm of the spectral ratio and the frequency is linear,

$$\ln\left[\frac{S(f)}{R(f)}\right] = \left(\frac{\pi d}{v_p Q}\right) f + \ln G, \tag{26}$$

where $f = \omega/(2\pi)$, and v_p is the phase velocity at the frequency f. Since in this case the wavelength is large compared to the layer thickness, the medium can be considered homogeneous, and the velocity can be estimated by dividing the source-receiver distance by the traveltime. Then, the quality factor Q can be determined from the slope of the line fitting $\ln\left[S(f)/R(f)\right]$.

On the other hand, the frequency-shift method (Quan and Harris, 1997) is based on the fact that the high frequencies attenuate more rapidly than the low frequencies. This effect may be quantified by measuring the resulting downshift $\Delta f = f_S - f_R$, where f_S and f_R are the centroid frequencies of $S(f)$ and $R(f)$, respectively. Then, if we approximate the spectrum $S(f)$ by a Gaussian with variance σ_s^2, we have

$$Q = \frac{\pi d \sigma_s^2}{v_p \Delta f}. \tag{27}$$

EXAMPLE

The 2-D stratified medium is composed of alternating plane layers of sandstone and limestone of equal thickness. The material properties are given in Table 1, where the relaxation frequency is $f_0 = 1/(2\pi\tau_0)$. Figure 1 shows the phase velocity (a), energy velocity (b) and quality factor (c) of the equivalent anisotropic medium at 25 Hz. The anisotropy coefficient is $Q_\epsilon = 0.66$.

The simulation uses 405×405 grid points and a vertical source. The grid spacing is 10 m and the source dominant frequency is 25 Hz. The dots in Figures 1b and 1c represents the results of the simulation.

CONCLUSIONS

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Table 1. Material properties

Medium	c_P (m/s)	c_S (m/s)	Q_{01}	Q_{02}	f_0 (Hz)	ρ (kg/m^3)
Sandstone	2950	1615	30	20	25	2300
Limestone	5440	3040	180	140	25	2700

$\tau_0 = 1/(2\pi f_0)$

FIGURES

FIG. 1. qP-wave phase velocity (a), energy velocity (b) and quality factor (c) of the equivalent anisotropic medium at 25 Hz. The dotted curve corresponds to the qS wave. The symbols in (b) and (c) represent the results of the numerical simulation.

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