CS 3510
Practice Mid Term
Time: 1hrs 20 min.

Write your name in the top left corner. Attempt all questions. You must show all work in order to obtain credit

1. (a) Order the following function in order of their asymptotic growths.
$n^{.01}, (\log n)^{10}, \log(n^{10}), 2^{\log_2 n}, 4^{\log_2 n}, n!, n^{\log n}, 2^n, n^n$.

   (b) Find a tight upper bound for the function $T(n)$ satisfying the recurrence,

$$T(n) = T(n/2) + T(n/4) + T(n/8) + O(n).$$

2. Let G be a graph on n vertices whose edges are inserted independently and randomly with probability p, i.e for any pair of distinct vertices u, v, the probability that $(u, v) \in E(G)$ is p. For any three distinct vertice u, v, w, we say that u, v, w form a triangle in the graph if $(u, v), (v, w), (w, u)$ are all in $E(G)$. What should be the value of p such that the expected number of triangles in the graph is at least 1 ?

3. Find an optimal parenthesization of a mtarix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$.

4. Write a complete proof of the path-relxation property in the context of single source shortest path algorithms. Recall that the path relaxation property states that if $p = \langle v_0, \ldots, v_k \rangle$ is a shortest path from $s = v_0$ to v_k, and the edges of p are relaxed in the order

$$(v_0, v_1), \ldots, (v_1, v_2), \ldots, \ldots, (v_{k-1}, v_k),$$

then $d[v_k] = \delta(s, v_k)$.

5. Let G be a weighted undirected graph with non-negative edge weights. We *define* the weight of a path $p = \langle v_0, \ldots, v_k \rangle$ to be $\max_{0 \le i < k} w(v_i, v_{i+1})$ (that is the weight of a path is the maximum of the weights of the edges appearing in the path). Write down an efficient algorithm for solving the all pairs shortest path problem with this new definition of weight of a path.