
1. Consider the law of composition
\[ x \circ y = (x^3 + y^3)^{1/3} \]
on the set \( \mathbb{R} \) of the real numbers. Prove that this law satisfies the group axioms.

2. Recall the definition of the dihedral group
\[ D_8 = \{ e, \sigma, \sigma^2, \sigma^3, \rho, \sigma \rho, \sigma^2 \rho, \sigma^3 \rho \} \]
discussed in class on Fri, Sept 15. List all subgroups of \( D_8 \) (you do not need to prove that these are all the subgroups).

3. Two elements \( x, y \) of a group \( G \) are said to be conjugate in \( G \) if there exists an element \( s \in G \) such that \( y = sxs^{-1} \). Show that the relation ” \( x \) and \( y \) are conjugate” is an equivalence relation on the set \( G \).

4. Let \( G \) be the set of all \( 2 \times 2 \) real matrices
\[
\begin{bmatrix}
a & b \\
0 & d \\
\end{bmatrix}
\]
with \( ad \neq 0 \). Prove that \( G \) forms a group under matrix multiplication. Is \( G \) abelian?

5. Recall the definition of subgroups from class. “A subset \( H \) of \( G \) is a subgroup if it contains the identity element, and is closed under taking products and inverses.” Prove that a non-empty subset \( H \) of a group \( G \) is a subgroup of \( G \) if for all \( a, b \in H \), \( ab^{-1} \in H \).

6. Prove that if in a group \( G \), every element is its own inverse, then \( G \) is abelian.