Revision Sheet for Final Exam, May 04 (Thus) 2.50-5.40 pm, Rm 240, Skiles

- Calculus Part.

1. You should be able to write down the Taylor polynomials centered at 0 and also at any other value a i.e. Taylor series expansion in powers of $(x - a)$.
2. You should know both the integral form and the Lagrange form of the remainder and be able to use them to bound the error. Most importantly, be sure that you understand how to compute the degree of the Taylor polynomial required in order for the error to be smaller than some given amount.
3. You should know the Taylor expansions of some familiar functions – that is you should be able to recognize them instantly.
4. The proof of Taylor's Theorem (Theorem 11.5.1) *is included*. DO NOT SKIP IT.
5. You should be able to use the L'Hopital's rule. Pay particular attention to indeterminate forms which are not in the standard $0/0$ or ∞/∞ form.
6. Make sure you know how to evaluate improper integrals – they are of two types. The limits might be infinite or the function itself might blow up at some point. You should be able to set up the appropriate proper integral and then take limits.
7. You should be able to make simple substitutions and manipulate the Σ notation.
8. You should know the definition of convergence of a series via partial sums. You should definitely be able to derive the formula for the geometric series through partial sums.
9. You should be able to pick and use all the standard comparison tests – including the integral test and the limit comparison test – as well as the root and the ratio test. The proofs for the last two *are included*.
10. You should know the definition of absolute and conditional convergence, as well as the alternating series test. Proof of why the alternating series test works is included.
11. Finally, you should know the definitions of the radius of convergence, and the interval of convergence of a power series. You should be able to compute the interval of convergence, by first applying the root or the ratio test *and then separately testing the end points.*
12. You should be familiar with differentiating and integrating power series "term by term" (Theorems 11.8.2 and 11.8.3).
13. You should know the properties of the binomial series. For instance, you should be able to write down the series expansion of $(1 - 2x)^{-1/3}$ and its radius of convergence.
14. You should know what a first order linear differential equation is. Remember that the unknown you are solving for is a *function* not a number.

15. You should know how to solve an equation of the form,

$$y' + p(x)y = q(x)$$

using the integrating factor $e^{\int p(x)dx}$.

16. Recall what is a *general solution* and what is a *particular solution.* You should know how to obtain particular solutions from a the general solution using the given initial values.

17. Do not skip the important example of Netwon's law of cooling.

18. You should know when a first-order differential equation is separable. If it is not immediately separable, you should be able to manipulate the equation to put it in separable form if possible. What does separability buy you ? What are integral curves ?

19. Be sure to understand the important example of the *logistic equation.*

- Linear Algebra Part.

1. You should be able to associate a matrix to a given linear transformation and vice-versa. For example, what is the matrix corresponding to the linear transformation of "differentiation" which a polynomial of degree at most four to one of degree at most three, where the polynomials are represented by usual column vectors ? Make sure you understand that composition of linear transformations corresponds to matrix products.

2. You should know the basic definitions of matrix-vector, matrix-matrix products.

3. You should know the definition of the dot product and its geometric significance (in terms of the angle between two vectors).

4. What is the length of a vector in terms of the dot product ?

5. You should know and be in a position to apply the Schwartz, Minkowski and the triangle inequalities.

6. You should know the definition of an isometry. What is the characterizing property of a matrix corresponding to an isometry ?

7. What effect does a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$ have on areas ? What are the area preserving linear transformations ?

8. You should know how to represent systems of linear equations in matrix notation, using augmented matrices. When are two such systems equivalent ?

9. You should be able to solve linear systems of equations using row reductions and back substitutions. In particular, you should know the row reduction algorithm (Section 3.4) and be able to use it.

10. What is the kernel of a matrix ? You should be able to compute a parametrization of the kernel using row reductions.
11. What is the importance of the pivotal columns ? How are they related to the rank of a matrix ?
12. Make sure you understand *thoroughly* the statement *as well as* the proof of Theorem 7 (page 121) and its corollary (page 122). In particular, you should be able to use the corollary to prove that a given square matrix is invertible by showing that is kernel is $\{0\}$ (cf. next two items).
13. Why is a square isometry invertible ? What is its inverse ?
14. When is a Vandermonde matrix invertible ? The proof is included.
15. You should be able to compute inverses of matrices using an extension of the row reduction algorithm.
16. What is the LU factorization of a matrix ?
17. What is the image of a matrix A ? Given a matrix A you should be able to compute equations for the image of A – that is express $\mathrm{Img}(A)$ as the kernel of some matrix.
18. You should also be able to parametrize the image of a matrix A using the pivotal columns.
19. You should be able to solve the closest vector problem using normal equations.
20. What are subspaces ? Make sure you understand why the image as well as the kernel of a matrix are examples of subspaces. Why is the integer lattice not a subspace ? Is the line defined by $X + Y = 1$ a subspace of $\mathbb{R}^2$?
21. What is the span of a given set of vectors ? The image of a matrix is the span of its column vectors.
22. When is a set of vectors linearly independent ? Theorem 5 (page 179) is very important. Make sure you understand its statement, as well as the proof.
23. What is the basis of a subspace ? You should be able to compute basis vectors for the image and kernel of any given matrix.
24. What are co-ordinates of a vector with respect to a given basis ? How do you calculate them ? Why is it easier in the case the given basis is orthonormal ?
25. Make sure you know the statement as well as the proof of Theorem 8 (page 187).
26. What is the dimension of a subspace ? Make sure you understand that the rank of a matrix is the dimension of its image. What is the nullity of a matrix ? What is the sum of the rank and the nullity of a matrix ?
27. What is an orthogonal projection ? What is the orthogonal complement of a subspace ?

28. You should know how to compute Gram-Schmidt orthogonalizatio and QR-factorization.

If you know all of the above you will do fine in the test. GOOD LUCK!