Solutions to Hwk7

Section 6.2

24.

\[
\begin{array}{c|ccccc}
    x & f(x) \\
    \hline
    4 & 63 & 26 & 6 & 1 \\
    2 & 11 & 2 & 5 \\
    0 & 7 & 7 \\
    3 & 28 \\
\end{array}
\]

Thus, \( p(x) = 63 + 26(x - 4) + 6(x - 4)(x - 2) + x(x - 4)(x - 2). \)

Section 6.3

1.

\[
\begin{array}{c|ccccc}
    x & p(x) \\
    \hline
    0 & 2 & -9 & 3 & 7 & 5 \\
    0 & 2 & -6 & 10 & 17 \\
    1 & -4 & 4 & 44 \\
    1 & -4 & 48 \\
    2 & 44 \\
\end{array}
\]

So \( p(x) = 2 - 9x + 3x^2 + 7x^2(x - 1) + 5x^2(x - 1)^2. \)

3. By Theorem 1, there exists a unique polynomial \( p \) of degree \( \leq m \) (\( m = 2n + 1 \)) such that \( p(x_i) = y_i \) and \( p'(x_i) = 0 \) for \( 0 \leq i \leq n. \) By Equation (9)

\[
p(x) = \sum_{i=0}^{n} y_i [1 - 2(x - x_i)\ell'_i(x_i)]\ell_i^2(x)
\]

where \( \ell_i(x) = \prod_{j=0, j\neq i}^{n} (x - x_j)/(x_i - x_j) \) for \( 0 \leq i \leq n. \)

4. Let us write \( p(x) = a + b(x - x_0) + c(x - x_0)^2 + d(x - x_0)^3. \) Then \( p'''(x) = 2c + 6d(x - x_0). \)

The four conditions can be written as: \( c_{00} = p(x_0) = a, \ c_{02} = p''(x_0) = 2c, \)

\( c_{10} = p(x_1) = a + bh + ch^2 + dh^3, \) and \( c_{12} = p'''(x_1) = 2c + 6dh \) when \( h = x_1 - x_0. \) So \( a \) and \( c \) are obtained without restrictions: \( a = c_{00}, \ c = c_{02}/2. \) \( d \) and \( b \) can be obtained from last two equations: \[
\begin{bmatrix}
    h & h^3 \\
    0 & 6h
\end{bmatrix}
\]

is known vector. \( \det \begin{bmatrix}
    h & h^3 \\
    0 & 6h
\end{bmatrix} = 6h^2 \neq 0 \) iff \( h \neq 0 \)

\( \Rightarrow \) condition: \( x_0 \neq x_1. \)