Evidence for internal structures of spiral turbulence

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We present several observations into spiral turbulence in a Taylor-Couette geometry gained through a three-dimensional direct numerical simulation. Conditionally averaged flow statistics show the persistence of an azimuthal gradient of the mean flow across both the turbulent and laminar spirals, and distinct distribution features of the turbulent intensity. The data provide a physical picture qualitatively different from the existing model of spiral turbulence. Certain aspects of the spiral pattern are observed to bear similarities to the stationary laminar-turbulent pattern in plane Couette flow.

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The coexistence of turbulent and laminar domains in space and time is one of the most fascinating phenomena in fluid dynamics. Spatiotemporal intermittency and pattern formation in such flows have been observed for a variety of systems [1]. Particularly intriguing is the spiral turbulence regime (barber-pole pattern) in the Taylor-Couette setting where intertwined helical turbulent and laminar stripes propagate between counter-rotating concentric cylinders [2,3]. On the largest scale spiral turbulence is observed to relate to a finite-wavelength modulation of turbulent intensities, and aspects of the spiral pattern can be qualitatively described by model equations [3]. In other systems related patterns are the stationary laminar-turbulent pattern in plane Couette flow and the torsional flow between a stationary and a rotating disk [4].

In this Brief Report we present several observations into spiral turbulence gained from a three-dimensional (3D) direct numerical simulation. By employing conditional averaging techniques, we have obtained flow statistics of a single turbulent or laminar spiral for a range of Reynolds numbers. The data demonstrate large-scale spatial variations of the flow internal to the turbulent and laminar spirals, and unique characteristics in the mean flow and turbulent intensity.

We consider the incompressible flow between two concentric cylinders with periodic boundary conditions in the axial direction. The cylinder axis is aligned with the $z$ axis of the coordinate system. The geometry is characterized by the radius ratio $\eta=R_i/R_o$ ($R_i$ and $R_o$ are, respectively, the inner- and outer-cylinder radii) and the aspect ratio $\Gamma=L_z/d$ ($L_z$ is the domain axial dimension and $d$ is the gap width, with $d=R_o-R_i$). The inner cylinder rotates counterclockwise (viewing toward the $-z$ direction) at an angular velocity $\Omega_i$, and the outer cylinder rotates clockwise at an angular velocity $\Omega_o$. All length variables are normalized by $R_i$, the velocity by a natural velocity $U_j$ (leading to $\Omega_i R_o U_j = -1.08$), and the pressure by $\rho U_j^2$ ($\rho$ is the fluid density). The inner- and outer-cylinder Reynolds numbers are defined by $Re_i=\Omega_i R_o d/\nu$ and $Re_o=\Omega_o R_o d/\nu$ ($\nu$ is the kinematic viscosity).

Our computational algorithms have been documented in detail in previous works [5]. In brief, we numerically solve the 3D Navier-Stokes equations employing a Fourier spectral expansion of flow variables along the axial direction and a high-order spectral-element discretization of the annular domain. The effectiveness of spectral-element-based approach has been evidenced in previous studies [6]. The time discretization is based on a stiffly stable scheme [7]. No-slip boundary conditions are applied on the inner and outer cylinders to reflect their rotation velocities.

We consider a radius ratio $\eta=0.89$, comparable to those of previous experiments [2,3]. We fix $Re_o$ at $-1375$ and vary $Re_i$ between 530 and 900. In light of the computational cost, we aim to simulate only one complete turbulent spiral. For this purpose we consider several aspect ratios ranging from $\Gamma=6$ to 25. At $\Gamma=12$ and higher we have observed complete turbulent spirals, while at a low $\Gamma$ no spiral can be observed. Results reported herein are for $\Gamma=25.1$. To ensure convergence of the simulation results, we have varied the resolutions systematically. The number of Fourier planes in the axial direction is varied between 384 and 512. In the annular domain 640 quadrilateral spectral elements are employed; and the element order is varied from 6 to 9. By comparing profiles of the time-averaged mean and root-mean-square (rms) fluctuation velocities at different resolutions, we have confirmed the convergence of our simulation results. Our application code has been extensively validated for Taylor-Couette turbulence by comparing the computed flow quantities with those determined from experiments; see [5] for details on the validations. We have also compared the turbulent fraction of spiral turbulence obtained from the current simulation and the experiment of Goharzadeh and Matabazi [2], and good agreement has been observed.

We start by exploring the Reynolds number dependence of the patterns. Figure 1(a) is a composite plot of a long simulation spanning $530 \leq Re_i \leq 900$, with $Re_i$ increased in discrete steps and $Re_o$ fixed at $-1375$. Shown are the stable patterns at each $Re_i$ (transients at the change in $Re_o$ are not shown). We record time histories of the velocity over points along a line parallel to the $z$ axis and fixed in the midgap. Plotted are the azimuthal velocity contours in spatial-temporal ($z$-$t$) plane. Distinct patterns can be identified with increasing $Re_i$. At $Re_i=530$ turbulent patches (bursts) are observed to emerge from the laminar background, persist for a while, and then disappear into the flow; some aspects of the turbulent bursts are described in [8]. At $Re_i=560$, we mostly observe complete spirals which appear quite regular; however, from time to time the spiral becomes broken, with two or more pieces. At $Re_i=611-700$, one can observe regular turbulent-laminar spirals, characterized by regularly spaced
To explore the statistical features of turbulent spirals, we study how the flow evolves statistically with changes in Re. At Re = 700, we do not observe both types of spiral patterns. For all Re ≥ 900 the spiral patterns can be clearly distinguished, and the apparent large-scale pattern can be discerned.

We next focus on Re values with well-defined spiral patterns. Figure 1(b) shows a typical turbulent-laminar spiral pattern from the simulation. We plot here the instantaneous azimuthal velocity contours in a grid surface showing the turbulent spiral at Re = 700. (c) Isosurface of conditionally averaged rms velocity magnitude u’/Ud = 0.14 (Re = 611).

Inclined stripes in Fig. 1(a). As Re increases to 750 and 800, the pattern becomes less recognizable, and turbulent fluctuations increasingly dominate the flow. At Re = 900 the entire flow becomes turbulent, and no apparent large-scale pattern can be discerned.

The other type is a spatiotemporal conditional averaging. It is applied to the spatial-temporal data [see Fig. 1(a)], which provides a velocity time history at each axial location z. The turbulent (laminar) phases in the velocity histories at different z’s are not aligned in time, thus leading to the inclined stripes in Fig. 1(a). A given Re, the average inclination angle of the stripe pattern in Fig. 1(a) is first determined, which provides the phase-shift information. The velocity history at a location z0 is considered as the base history. Then we shift in time the velocity history at any z to align the turbulent phases with the base history, based on (z−z0) and the average inclination angle. The shifted data are accumulated to the base velocity history for averaging. This results in a velocity history, conditionally averaged over the points along the z direction. It is a periodic signal, with the rotation period of the spiral pattern as its period. We then shift in time this conditionally averaged velocity history and average over different periods.

The conditionally averaged statistics indicate the existence of internal structures of the spiral turbulence. First, an azimuthal velocity gradient, ∂⟨uθ⟩/∂θ (⟨uθ⟩ denotes conditional mean azimuthal velocity, and r and θ are radial and azimuthal coordinates), persists across the turbulent and laminar spirals. Figure 2(a) shows several periods of the spatiotemporally averaged conditional mean and rms azimuthal velocity, together with an instantaneous base history at z0. Alternating turbulent and laminar phases can be clearly distinguished. Most striking is the large systematic variation of the conditional mean velocity in the turbulent and laminar phases. Note that the onset of turbulent phase corresponds to the passing of the leading edge of a turbulent spiral over a fixed point in space, and the end of turbulent phase corresponds to the passing of the trailing edge. Therefore, Fig. 2(a) indicates that along the azimuthal direction the mean velocity experiences a substantial variation across the turbulent and laminar spirals. The trailing edge of a turbulent spiral has a notably higher mean velocity magnitude than the leading one. It is the reverse for a laminar spiral. This observation applies to all Re studied here with well-defined spiral patterns.

### Figure 1

**Figure 1.** (Color online) Turbulent-laminar patterns. (a) Contours of azimuthal velocity in spatial (z) and temporal (t) planes. Time evolves horizontally with changes in Re, indicated at the top. Dark and blank regions, respectively, represent turbulent and laminar flows. (b) Instantaneous azimuthal velocity contours in a cylindrical grid surface showing the turbulent spiral at Re = 700. (c) Iso-surface of conditionally averaged rms velocity magnitude u’/Ud = 0.14 (Re = 611).
This observation has a connection to the Hayot and Pomeau model [9] of spiral turbulence. When modeling the coexistence of stable laminar and turbulent domains of spiral turbulence using the Ginzburg-Landau equation, Hayot and Pomeau introduced a crucial nonlocal term, which leads to a stable domain structure, whereas without such a term no stable domains coexist. The model is based on two crucial assumptions: (1) the azimuthal dependence of the mean azimuthal velocity and (2) the presence of a mean azimuthal pressure gradient. It concludes that (1) the mean azimuthal pressure gradient has large magnitudes in both the laminar and turbulent regions and (2) a large Poiseuille flow component is present in the mean azimuthal velocity due to the mean pressure gradient. The large Poiseuille component has also been argued by Hegseth et al. [3].

Our observation above is consistent with the first assumption that underpins the Hayot and Pomeau model. However, simulation results provide a physical picture qualitatively different than the model. To examine the pressure, we show in Fig. 2(b) the spatiotemporal conditional mean and rms pressure, and the instantaneous pressure history at $z_0$. The conditional mean pressure is essentially constant in the laminar phase and exhibits notable variations only in the turbulent phase. That is, the mean pressure is essentially constant across the laminar spiral and has a significant azimuthal gradient only in the turbulent spiral region. This is very different from the conclusion of the Hayot and Pomeau model. Figure 2(c) shows profiles (across the cylinder gap) of the conditionally averaged mean azimuthal velocity at several azimuthal locations: the leading edge, trailing edge, and the core of the turbulent spiral, as well as the core of the laminar spiral. Examination of these mean velocity profiles shows that the variation of the mean azimuthal velocity along the azimuthal direction has a rather involved characteristic and is not of Poiseuille flow type. Significant changes in the mean velocity tend to occur in only part of the cylinder gap. For example, from the turbulent spiral core to the trailing edge the mean azimuthal velocity has a significant increase in magnitude in the outer half of the gap, but essentially no change toward the inner half, while from the leading edge to the turbulent core the largest change occurs in the inner portion of the gap. Therefore, our simulation results suggest that significant mean azimuthal pressure gradient exists only in the turbulent spiral region, and that the azimuthal Poiseuille flow component from the Hayot and Pomeau model is not evident.

The second main observation about spiral turbulence is that the cores of turbulent and laminar spirals are demarcations of axially opposite flows in the mean sense. The mean axial flow tends to be away from the core of a turbulent spiral and toward the cores of adjacent laminar spirals. This is demonstrated in Figs. 3(a) and 3(b), which, respectively, show contours of the conditional mean axial velocity and conditional rms velocity magnitude $u'/U_2$ in a radial-axial plane. One can observe that the core of the turbulent spiral marks an interface; on both sides the flow tends to be away from this interface. Similarly, the laminar spiral core marks another interface at which flows on both sides tend to be
The strongest turbulent intensity tends to be located near both toward each other. This observation is generic to all Re studied here with both left- and right-handed spirals. It is, however, somewhat counterintuitive. For example, because a left-handed turbulent spiral propagates axially along the $-z$ direction, intuitively the leading and trailing edges would have a negative mean axial velocity. This is true for the leading edge. Contrary to intuition, however, the trailing edge has actually a positive mean axial velocity. A similar situation occurs to right-handed spirals.

Third, the distribution of turbulent intensity exhibits a different characteristic in spiral turbulence than in fully developed turbulence. This is demonstrated in Figs. 3(b) and 3(c) which show contours of $u'/U_d$ in a horizontal $x,y$ plane at midheight of the cylinder. In spiral turbulence the most energetic intensity appears at the core of turbulent spiral, toward the middle of the gap. In contrast, in fully developed Taylor-Couette turbulence at high Reynolds numbers the strongest turbulent intensity tends to be located near both walls rather than in the midgap [5]. Figures 3(b) and 3(c) also clearly illustrate our previous point that the leading edge of the turbulent spiral has a proximity to the outer wall while the trailing edge has a proximity to the inner one, a point also noted by Atta [2].

The observations discussed above for spiral turbulence can be contrasted with, and oftentimes, can find their counterparts in the stationary turbulent-laminar pattern in plane Couette flow. Here, we refer to the work of Barkley and Tuckerman [10], which provides a detailed analysis of the mean flow and force balance of the patterns in plane Couette flow. The negligible mean pressure gradient in laminar spiral region [Fig. 2(b)], the phase difference between mean and rms velocities as shown in Fig. 2(a), and the sense of mean axial flow relative to spiral cores [Fig. 3(a)] are consistent with the data for plane Couette flow [10]. The similarity between spiral turbulence and the pattern in plane Couette flow is noted by several studies (see Prigent et al. [3], among others).

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