Project 1 (Due March 11)

1 Your personal number

The parameter s in the following problem is determined from the letters of your name in the following way. Let F1, F2 be the encoded values of the first two letters of your main forename and S1, S2 the encoded values of the first two letters of your surname using the code $A = 1$, $B = 2, \ldots$, $Z = 26$. Then

$$s = 0.6 + 0.0001(125S1 + 25F1 + 5S2 + F2).$$

For example, ALEX MCLEISH has A and L as the first two numbers of his forename giving $F1 = 1$, $F2 = 12$. Similarly, M and C of his surname give $S1 = 13$ and $S2 = 3$. Hence

$$s = 0.6 + 0.0001(12513 + 251 + 53 + 12) = 0.7677.$$

The function file **pernum.m** will calculate your personal number to 4 decimal places. For example if ALEX MCLEISH wanted his number he would type

```
s = pernum(1, 12, 13, 3)
```

which would return the value $s = 0.7677$. Note that your personal number should lie in the range $0.6 < s < 1$.

2 The problem

This project is concerned with the application of Adams-Bashforth-Moulton predictor-corrector methods to solve the problem

$$y' = \frac{y}{t} - s(\frac{y}{t})^2,\ 1 \leq t \leq 4,\ y(1) = 1,$$

where s is your personal number. The exact solution is

$$y(t) = \frac{t}{slog(t) + 1}.$$

You will need the predictor-corrector code **abmpc.m.** To run the program type

```
[t,y,trunc,mil]=abmpc(a,b,eta,N,iex,k,m,mode)
```

the MATLAB prompt in the command window, where *a, b, eta, N, iex, k, m, mode* are the parameters you which to input.

- $[a, b]$ is the time interval.
- $eta = y(a)$ is the initial value and N is the number of time steps.
- If $iex = 1$, the exact solution is used to provide the necessary starting values. Otherwise, you will need to input starting values.
- The parameter k is the step-number of the explicit predictor. So for example, if we want to use the AB2 method which uses the Adams-Bashforth 2-step predictor then set $k = 2$.
- The parameter m is the number of correction steps that are carried out. Finally, the code will operate in $P(EC)^m$ mode when $mode = 0$ and $P(EC)^m E$ mode when $mode = 1$.

Output from the code is the numerical solution y, the local truncation error **trunc** and Milne's estimate **mil**. Remember to edit the programs **f.m** and **yexact.m** to correspond to the functions above and remember s is your personal number.

Perform the following using the code **abmpc.m**.

1. Calculate the numerical solution using the AB2 method with N = 32 in PEC mode. Plot the numerical and exact solutions on the one graph.
2. In a separate figure plot the absolute value of the error. On this same graph plot the truncation error and Milnes' estimate. Comment on what you observe.
3. Repeat the calculation above in P(EC)E mode. Do you find an improvement in the solution ?
4. Repeat the calculation above in $P(EC)^2E$ and $P(EC)^2$ modes. Comment on your results.

Improve/expand the code **abmpc.m** as follows:

1. Provide an automatic start-up procedure.
2. Add the necessary constants for $k = 5$ and $k = 6$.
3. Use Milnes' estimate to design an adaptive strategy (halving or doubling the time steps as in Section 4.8).
4. Check your improvement/expansion with the above example and comment on your results.