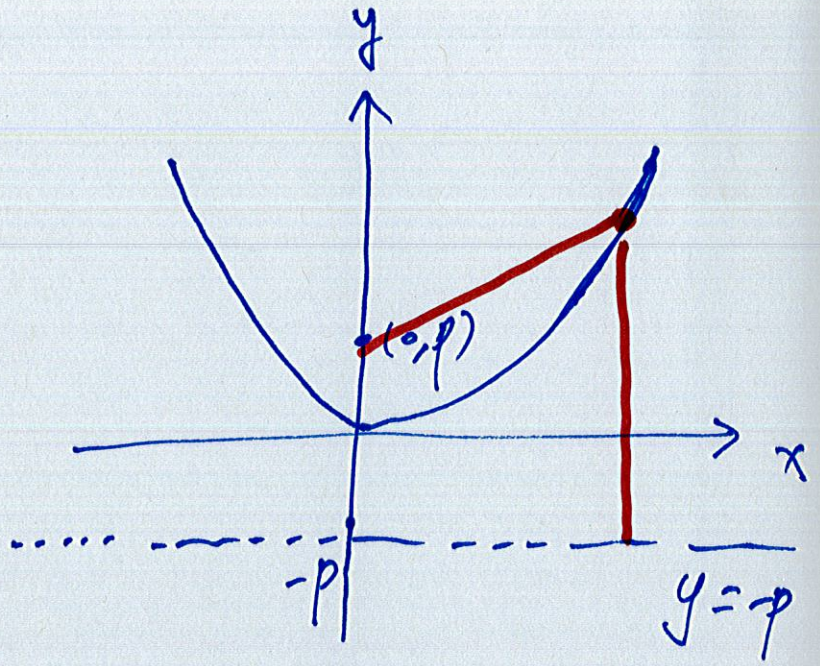
10.5 Continued ...

Recall:

Parabolas

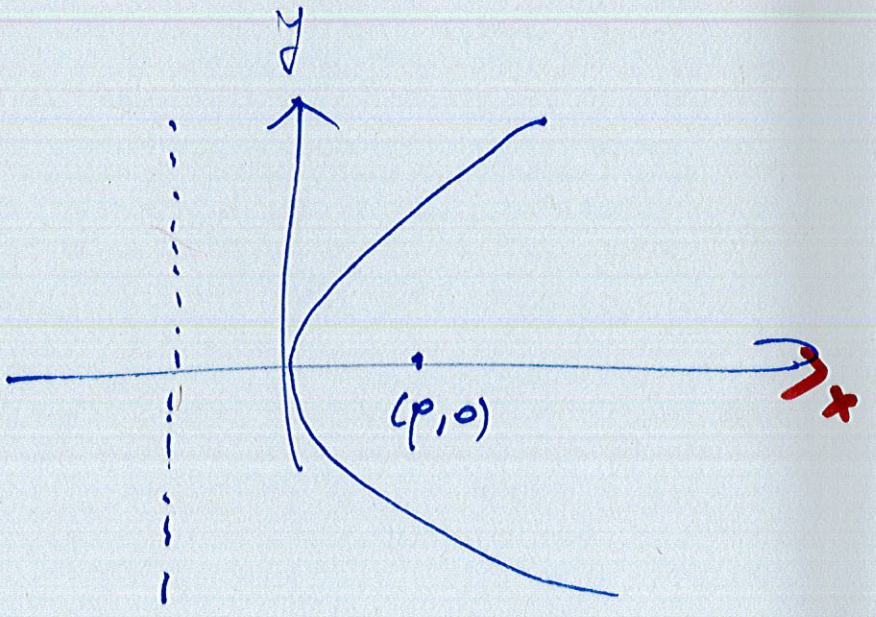
* $y = \frac{1}{4p}x^2$



Focus: $(0, p)$

directrix $y = -p$

* $x = \frac{1}{4p}y^2$

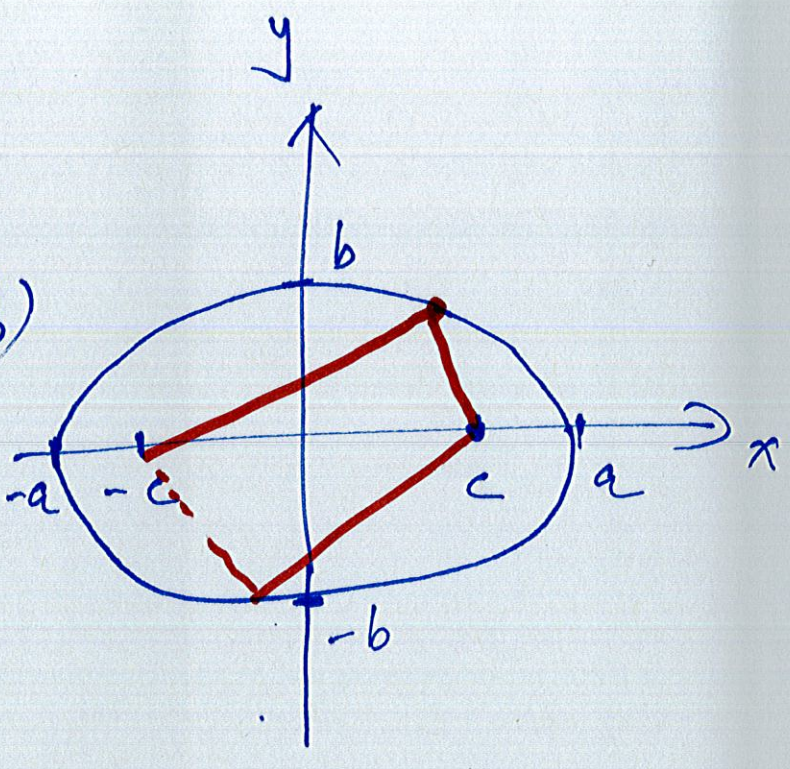


Focus $(p, 0)$

directrix $x = -p$

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a \geq b)$$



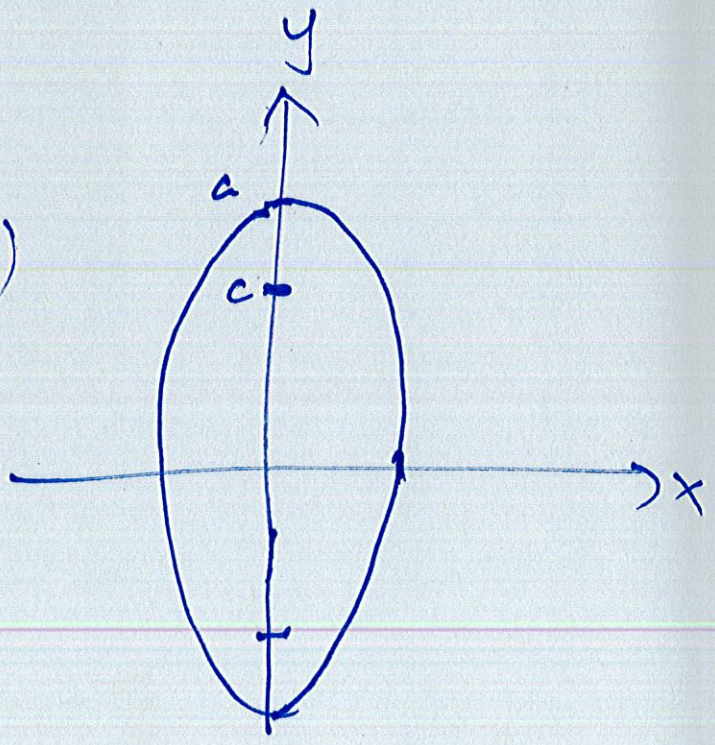
$$c^2 = a^2 - b^2$$

Foci : $(\pm c, 0)$

~~Vertices~~

vertex : $(\pm a, 0)$

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \quad (a \geq b)$$



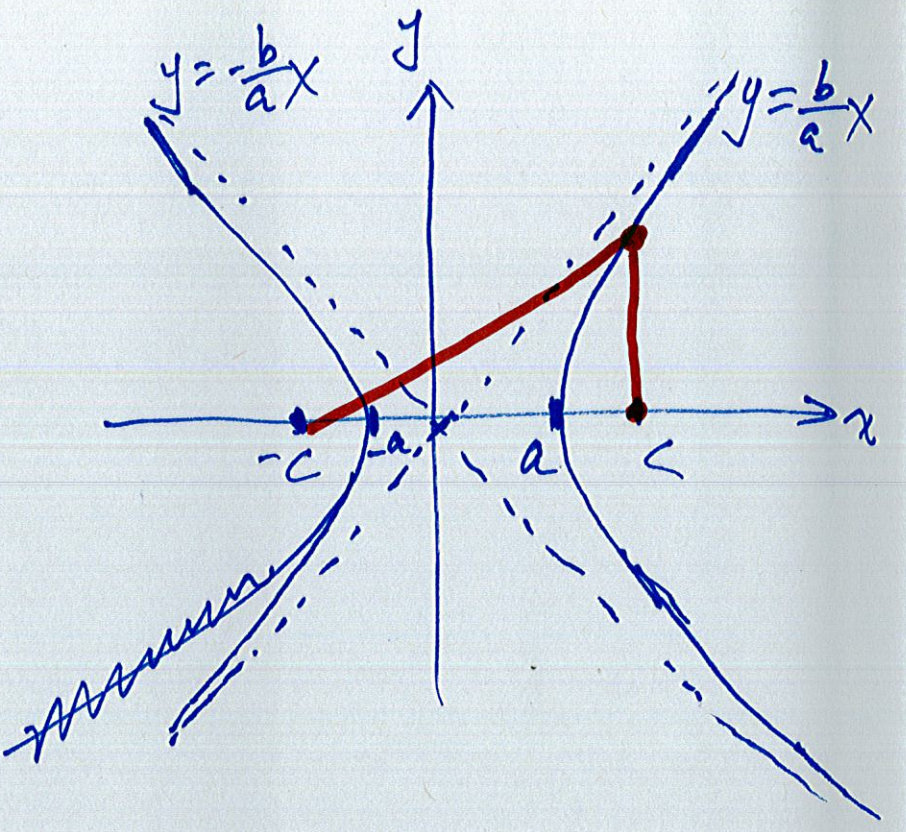
Foci : $(0, \pm c)$

$$c^2 = a^2 - b^2$$

Vertex : $(0, \pm a)$

Hyperbolas

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\left(\frac{x}{a} - \frac{y}{b}\right)\left(\frac{x}{a} + \frac{y}{b}\right) = 1$$

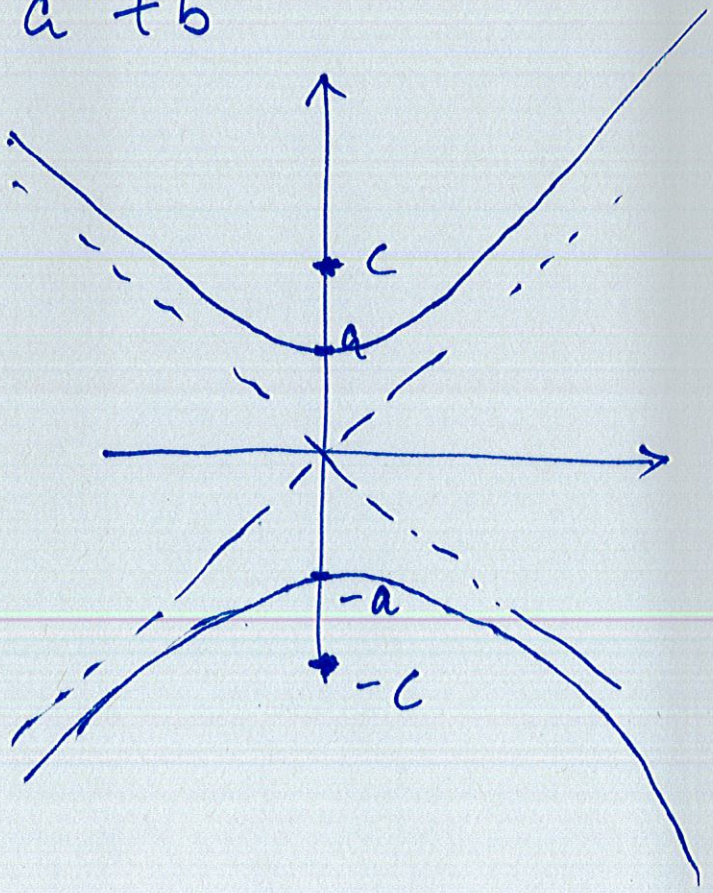
vertices: $(\pm a, 0)$

two asymptotes:

$$y = \pm\frac{b}{a}x \iff \frac{x}{a} \pm \frac{y}{b} = 0$$

Foci: $(\pm c, 0)$: $c^2 = a^2 + b^2$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



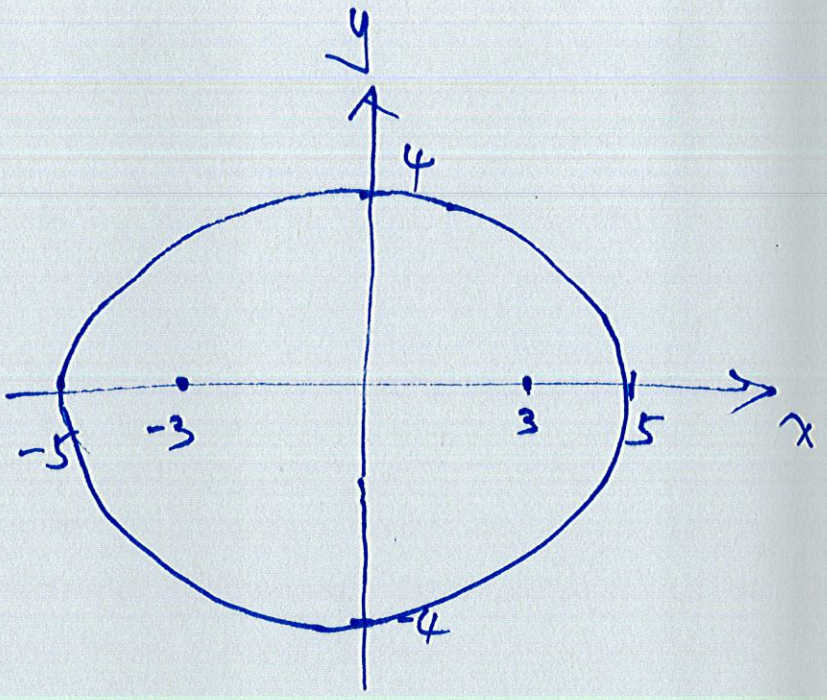
vertices: $(0, \pm a)$

Two asymptotes: $x = \pm\frac{b}{a}y$

Foci: $(0, \pm c)$

with $c^2 = a^2 + b^2$

Ex. The ellipse has vertices at $(\pm 5, 0)$ and Foci at $(\pm 3, 0)$



$a = 5$

$c = 3$

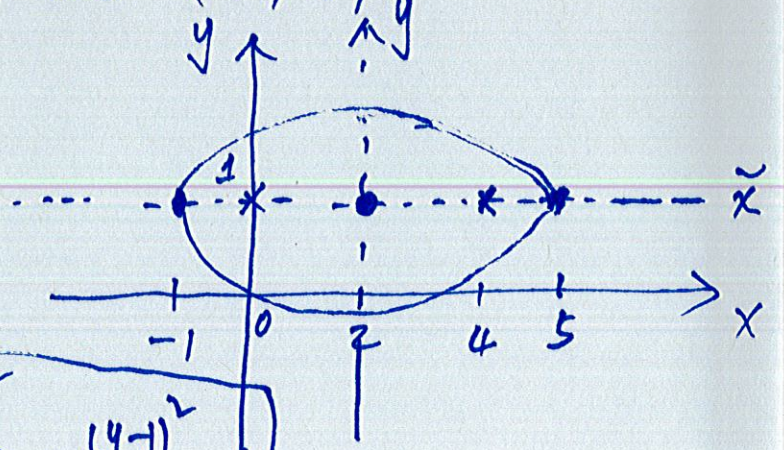
$b^2 = a^2 - c^2$

$= 25 - 9 = 16$

$\Rightarrow b = 4$

$$\Rightarrow \frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$

Ex. The ellipse has vertices at $(5, 1), (-1, 1)$ and Foci at $(4, 1)$ and $(0, 1)$



$a = 3$

$c = 2$

$b^2 = a^2 - c^2 = 5$

$$\frac{\tilde{x}^2}{3^2} + \frac{\tilde{y}^2}{\sqrt{5}^2} = 1 \Rightarrow \boxed{\frac{(x-2)^2}{3^2} + \frac{(y-1)^2}{\sqrt{5}^2} = 1}$$

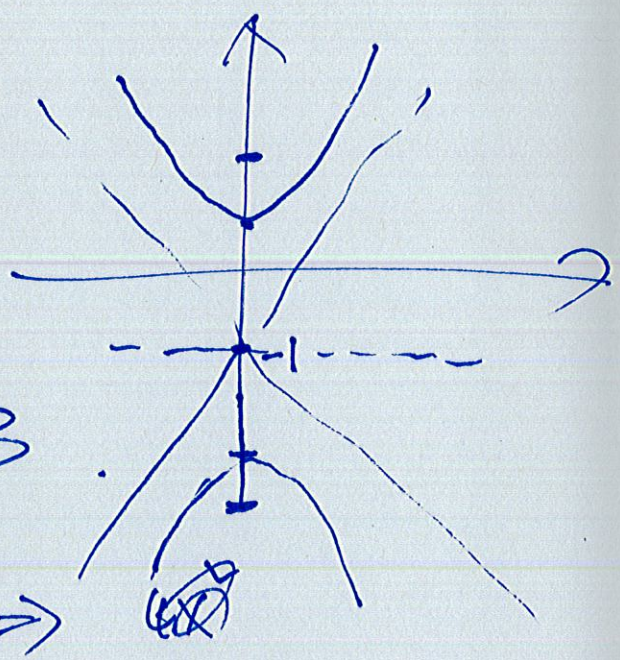
If the center is shifted to (x_0, y_0)

then, the equations should be shifted to:

$$\frac{(x-x_0)^2}{a^2} \pm \frac{(y-y_0)^2}{b^2} = 1$$

or

$$y - y_0 = \frac{1}{4p}(x-x_0)^2$$



Ex. $y^2 + 2y = 4x^2 + 3$

$$(y+1)^2 - 1 = 4x^2 + 3 \Rightarrow$$

$$(y+1)^2 - 4x^2 = 4 \Rightarrow \frac{(y+1)^2}{2^2} - \frac{x^2}{1^2} = 1$$

hyperbola: in the $(\tilde{x}, \tilde{y})$ =

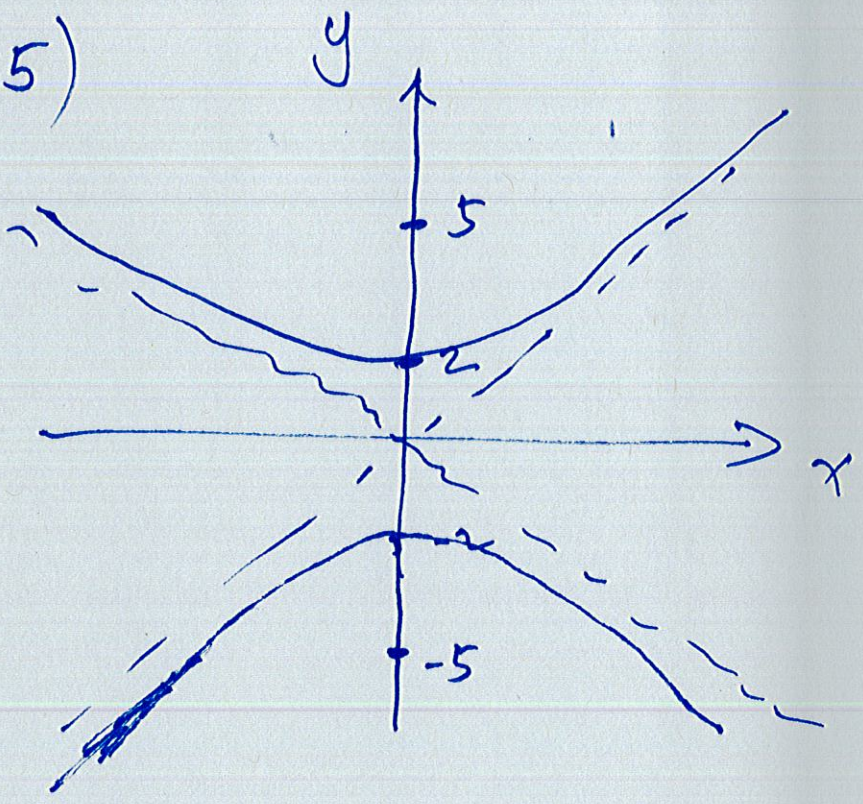
with vertices: $(0, \pm 2)$ $\quad c = \sqrt{2^2+1^2} = \sqrt{5}$

Foci: $(\tilde{x}, \tilde{y}) = (0, \pm\sqrt{5})$

$$\begin{cases} \tilde{x} = x \\ \tilde{y} = y+1 \end{cases}$$

Ex. Find the hyperbolas with vertices $(0, \pm 2)$ and foci $(0, \pm 5)$

eqn for the



$a = 2$

$c = 5$

$b^2 = c^2 - a^2$

$= 5^2 - 2^2 = 21$

$b = \sqrt{21}$

$$\boxed{\frac{y^2}{2^2} - \frac{x^2}{\sqrt{21}^2} = 1}$$

Two asymptots: $x = \pm \frac{b}{a} y = \pm \frac{\sqrt{21}}{2} y$

Ex. $9x^2 - 4y^2 - 72x + 8y + 176 = 0$

$9(x^2 - 8x) - 4(y^2 - 2y) + 176 = 0$

$9[(x-4)^2 - 16] - 4[(y-1)^2 - 1] + 176 = 0$

$9(x-4)^2 - 4(y-1)^2 = -36$

$$\Rightarrow \boxed{\frac{(y-1)^2}{9} - \frac{(x-4)^2}{4} = 1}$$

$a = 3$

$b = 2$

$c = \sqrt{a^2 + b^2} = \sqrt{13}$