MA 162 FINAL EXAM PRACTICE PROBLEMS Fall 2011

$\vec{v}\cdot\vec{w} = |\vec{v}||\vec{w}|\cos\theta$

1. Find the angle between the vectors $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

A. $\cos^{-1}\left(\frac{8}{9}\right)$ B. $\cos^{-1}\left(\frac{5}{9}\right)$ C. $\cos^{-1}\left(\frac{2}{3}\right)$ D. $\cos^{-1}\left(\frac{7}{9}\right)$ E. $\cos^{-1}\left(\frac{1}{3}\right)$

2. Find a such that $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + a\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ are perpendicular. you need $\vec{u}\cdot\vec{v} = 0$

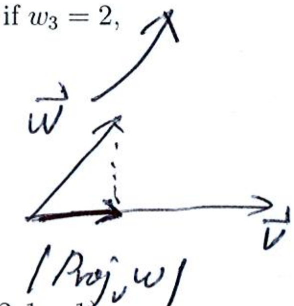
A. 3 B. 2 C. 1 D. -1 E. -2

3. If $\mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k}$ is perpendicular to $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, and if $w_3 = 2$, then $w_1 =$

A. 4 B. 2 C. -2 D. -4 E. 1

4. If $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{k}$, find $|\text{proj}_{\mathbf{v}}(\mathbf{w})|$. $= \left| \frac{\vec{v}\cdot\vec{w}}{|\vec{v}|^2}\vec{v} \right|$

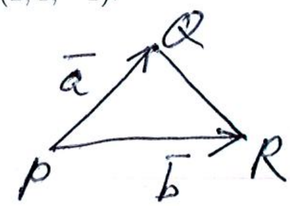
A. $1/\sqrt{3}$ B. $\sqrt{3}$ C. $\sqrt{3}/5$ D. $2\sqrt{3}$ E. $\sqrt{3}/2$



5. Find the area of the triangle with vertices $P = (0, 0, 0)$, $Q = (1, 2, 1)$, and $R = (2, 1, -1)$.

$A = \frac{1}{2}|\bar{a} \times \bar{b}|$

A. $\sqrt{27}$ B. $\frac{\sqrt{27}}{2}$ C. $\frac{\sqrt{11}}{2}$ D. $\sqrt{19}$ E. $\frac{\sqrt{3}}{2}$



6. The radius of the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z = 3$ is

complete the square →

A. $3 + \sqrt{13}$ B. $\sqrt{13}$ C. $\sqrt{65}$ D. $3 + \sqrt{56}$ E. $\sqrt{17}$

7. The area of the region enclosed by the curves $y = x^2 + 1$ and $y = 2x + 9$ is given by

$A = \int_a^b (f(x) - g(x))\,dx$

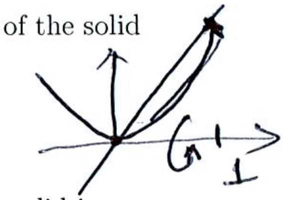
A. $\int_{-2}^{4}(x^2 + 1 - 2x - 9)\,dx$ B. $\int_{-2}^{4}(2x + 9 - x^2 - 1)\,dx$ C. $\int_{-2}^{2}(2x + 9 - x^2 - 1)\,dx$ D. $\int_{-4}^{2}(2x + 9 - x^2 - 1)\,dx$ E. $\int_{-4}^{2}(x^2 + 1 - 2x - 9)\,dx$

$x^2+1 = 2x+9$ to get a, b.

8. Let R be the region between the graphs of $y = x^2$ and $y = x$. Find the volume of the solid generated by revolving R about the x–axis.

$V = \int_0^1 \pi\left(x^2 - (x^2)^2\right)dx$

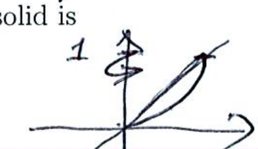
A. $\frac{\pi}{6}$ B. $\frac{\pi}{12}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{15}$ E. $\frac{2\pi}{15}$



9. If the region in problem 8 is revolved about the y–axis, then the volume of the solid is

$V = \int_0^1 2\pi(x - x^2)\,x\,dx$

A. $\frac{\pi}{6}$ B. $\frac{\pi}{12}$ C. $\frac{\pi}{24}$ D. $\frac{2\pi}{15}$ E. $\frac{\pi}{15}$

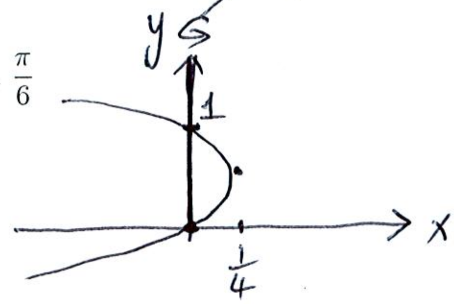


10. If R is the region bounded by the curves $x = 0$ and $x = y - y^2$, and if R is revolved around the y–axis, then the volume of the solid is

$V = \int_0^1 \pi(y - y^2)^2\,dy$.

A. $\frac{\pi}{15}$ B. $\frac{\pi}{30}$ C. $\frac{\pi}{12}$ D. $\frac{\pi}{3}$ E. $\frac{\pi}{6}$

$(y - \frac{1}{2})^2 = -x + \frac{1}{4}$



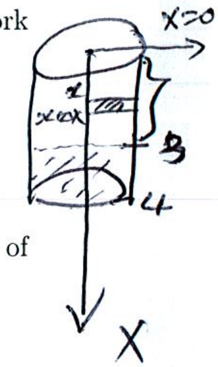
Hooke's Law: $F = kL \Rightarrow 4 = k\frac{1}{2} \Rightarrow k = 8$

11. A force of 4 lb. is required to stretch a spring 1/2 ft. beyond its natural length. How much work is required to stretch the spring from its natural length to 2 ft.

A. 8 ft–lbs. B. 12 ft–lbs. C. 16 ft–lbs. D. 24 ft–lbs. E. 32 ft–lbs.

12. A cylindrical tank of height 4 feet and radius 1 foot is filled with water. How much work is required to pump all but 1 foot of water out of the tank. (Density = 62.5 lbs./ft^3)

A. $9\pi(62.5)$ ft-lbs. B. $3\pi(62.5)$ ft-lbs. C. $\frac{9\pi}{2}(62.5)$ ft-lbs. D. $18\pi(62.5)$ ft-lbs.
E. $6\pi(62.5)$ ft-lbs.



13. Let $f(x) = \sqrt{x}$. Find c in $[0, 9]$ such that $f(c) = f_{\text{avg}}$, where f_{avg} is the average value of $f(x) = \sqrt{x}$ on the interval $[0, 9]$.

$f_{avg} = \frac{1}{9}\int_0^9 f(x)\,dx = \frac{1}{9}\int_0^9 \sqrt{x}\,dx$

A. $c = 4$ B. $c = 4.5$ C. $c = 5$ D. $c = 3.2$ E. $c = 6$.

14. $\underbrace{\int x(\ln x)^3 dx}_{\int u\,dv} = \underbrace{\frac{x^2}{2}}_{v}\underbrace{(\ln x)^3}_{u} - I$, where $I = \int v\,du$ Then, solve $\sqrt{x} = f_{avg}$.

A. $\frac{1}{4}\int(\ln x)^4 dx$ B. $\frac{1}{3}\int(\ln x)^2 dx$ C. $\frac{1}{3}\int(\ln x)^2 dx$ D. $\frac{3}{2}\int x^2(\ln x)^2 dx$ E. $\frac{3}{2}\int x(\ln x)^2 dx$

15. Evaluate $\int_0^1 \underbrace{x}_{u} e^{3x} dx$. $\frac{e^{3x}dx = dv}{v = \frac{1}{3}e^{3x}}$ $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$

A. $\frac{2e^3}{9}$ B. $\frac{1}{9}+\frac{2e^3}{9}$ C. 1 D. $\frac{1}{9}$ E. $\frac{e^3}{9} - 1$

16. $\int_0^{\pi/2} \sin^3 x dx =$ $\int (1-\cos^2 x)\, d(-\cos x)$ $\underline{u = \cos x}$ ... use $\sin^2 x + \cos^2 x = 1$

A. 2/3 B. 4/3 C. 0 D. 1/4 E. 1/3

17. $\int_0^{\pi/4} \underline{\sec^4 x} \tan x\, dx =$ $\overset{u=\tan x}{\int}$ $(\cancel{1+\tan^2 x})$ use $\frac{d\tan x}{dx} = \sec^2 x$ & $\sec^2 x = 1 + \tan^2 x$

A. 1 B. 1/3 C. 4/3 D. 3/4 E. 2/9

18. In order to compute $\int \frac{dx}{(1+x^2)^{3/2}}$ we make the substitution $x = \tan\theta$. This gives an integral in θ whose value is

A. $\frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C$ B. $\ln(\sec^2\theta) + C$ C. $\frac{1}{2}\theta + \tan^{-1}\theta + C$ D. $\frac{1}{2}\sqrt{\cos\theta} + C$
E. $\sin\theta + C$

$2x = 3\sin\theta$

19. $\displaystyle\int \frac{dx}{\sqrt{9-4x^2}} =$ $\cancel{2x=3}$ $\cancel{\displaystyle\int \frac{1}{\sqrt{1+x^2}}\,dx = \tan^{-1} x}$

A. $\sec^{-1}\left(\frac{3x}{2}\right)+C$ B. $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+C$ C. $\tan^{-1}\left(\frac{2x}{3}\right)+C$ D. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+C$
E. $\sqrt{9-4x^2}+\tan^{-1}\left(\frac{2x}{3}\right)+C$

20. $\displaystyle\int \frac{x+1}{x^3-2x^2+x}\,dx = \int \frac{x+1}{x(x-1)^2}\,dx = \int \frac{a}{x} + \frac{b}{x-1} + \frac{c}{(x-1)^2}\,dx$

A. $\ln|x|+\ln|x-1|+C$ B. $\ln|x|-\ln|x-1|+C$ C. $\ln|x|-\dfrac{2}{x-1}+C$

D. $\ln|x-1|-\dfrac{2}{x-1}+C$ E. $\ln|x|-\ln|x-1|-\dfrac{2}{x-1}+C$

21. A partial fraction decomposition of $\dfrac{x+2}{x^4+2x^2} = \dfrac{x+2}{x^2(x^2+2)}$ has the form

A. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+2}$ B. $\dfrac{A}{x^2}+\dfrac{Bx+C}{x^2+2}$ C. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^2+2}$ D. $\dfrac{A}{x^2}+\dfrac{B}{x^2+2}$
E. $\dfrac{A}{x}+\dfrac{B}{x^2+2}$

22. $\displaystyle\int_0^1 \frac{x+2}{x^2+1}\,dx = \int_0^1 \frac{x}{x^2+1}\,dx + 2\int_0^1 \frac{1}{x^2+1}\,dx$

A. $\dfrac{\ln 2}{2}+\dfrac{\pi}{2}$ B. $\dfrac{\ln 2}{2}$ C. $\dfrac{\ln 2}{2}+2\pi$ D. $2\ln 2+\dfrac{\pi}{2}$ E. $\ln 2+\pi$

$h = \frac{1}{3}$

23. Use the Trapezoidal Rule with $n=3$ to approximate $\displaystyle\int_0^1 \frac{1-x}{1+x}\,dx \simeq h\left[\frac{1}{2}\left(f(0)+f(1)\right)+f(\tfrac{1}{3})+f(\tfrac{2}{3})\right]$

$0 \quad \frac{1}{3} \quad \frac{2}{3} \quad 1$

A. $\dfrac{12}{5}$ B. $\dfrac{6}{5}$ C. $\dfrac{2}{5}$ D. $\dfrac{17}{60}$ E. $\dfrac{17}{10}$

24. Indicate convergence or divergence for each of the following improper integrals:

$$\text{(I)} \int_2^\infty \frac{1}{(x-1)^2}\,dx \qquad \text{(II)} \int_0^2 \frac{1}{(x-1)^2}\,dx \qquad \text{(III)} \int_0^1 \frac{\ln x}{x}\,dx = \lim_{\varepsilon\to 0}\int_\varepsilon^1 \frac{\ln x}{x}\,dx$$

$= \int_0^{1-\varepsilon} + \int_{1+\varepsilon}^2$ (as $\varepsilon \to 0$) $\qquad u = \ln x$

A. I converges, II and III diverge. B. II converges, I and III diverge. C. I and III converge, II diverges. D. I and II converge, III diverges. E. I, II and III diverge.

25. Find the length of the curve $y=\dfrac{2}{3}x^{3/2}$, $0 \le x \le 2$.

A. $2\sqrt{3}-2$ B. $3\sqrt{3}-1$ C. $\sqrt{3}-1$ D. $\dfrac{2}{3}\left(3\sqrt{3}-1\right)$ E. $3\sqrt{3}-2$

$$L = \int_0^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$