MA 514 HW 2

3.1 #5 Relative precision $\frac{|r-c_n|}{|r|} \leq \varepsilon$, since $r \geq a_0 > 0$.

We ~~have~~ need $\frac{|r-c_n|}{a_0} \leq \varepsilon$

By theorem: $|r-c_n| \leq 2^{-(n+1)}(b_0-a_0)$

$\therefore \quad 2^{-(n+1)}(b_0-a_0)/a_0 \leq \varepsilon$

$\therefore \quad -(n+1) \leq \frac{\log(\varepsilon a_0/(b_0-a_0))}{\log 2}$

$\therefore \quad n \geq \frac{\log(b_0-a_0) - \log\varepsilon - \log a_0}{\log 2} - 1$

3.2 #10 $f(x) = x^3 - R$, $f'(x) = 3x^2$

Newton's iteration formula is: $x_{n+1} = x_n - \frac{(x_n^3 - R)}{3x_n^2} = \frac{2x_n + R/x_n^2}{3}$

for $x>0$, $f'(x)>0$, $f''(x) = 6x > 0$ By thm 2, it will converge for any point >0

for $x<0$, Newton method will not always converge. for example,

$x_0 = \sqrt[3]{-\frac{R}{2}}$, $x_1 = 0$, $x_2 = \infty$, so the method fails here.

#15 $r = x_n - e_n \Rightarrow e_{n+1} = e_n - \frac{f(x_n)}{f'(x_0)}$

$f(r) = 0 \Rightarrow f(x_n - e_n) = 0 \Rightarrow f(x_n) - e_n f'(\xi_n) = 0$

$\therefore \quad e_{n+1} = e_n - e_n \frac{f'(\xi_n)}{f'(x_0)} = e_n\left(1 - \frac{f'(\xi_n)}{f'(x_0)}\right)$

$\therefore \quad s = 1 \qquad C = 1 - \frac{f'(\xi_n)}{f'(x_0)}$

3.2 #23(a) $f_1(x_1,x_2) = 4x_1^2 - x_2^2$, $f_2(x_1,x_2) = 4x_1x_2^2 - x_1 - 1$

$\therefore J = \begin{bmatrix} 8x_1 & -2x_2 \\ 4x_2^2-1 & 8x_1x_2 \end{bmatrix}$ ① $J(0,1) = \begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}$ $J^{-1}(0,1) = \frac{1}{6}\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$

$\begin{bmatrix} h_1^{(1)} \\ h_2^{(1)} \end{bmatrix} = -\frac{1}{6}\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix} \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{2} \end{bmatrix}$

$\therefore \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \end{bmatrix}$

② $J(\frac{1}{3},\frac{1}{2}) = \begin{bmatrix} \frac{8}{3} & -1 \\ 0 & \frac{4}{3} \end{bmatrix}$, $J^{-1}(\frac{1}{3},\frac{1}{2}) = \frac{9}{32}\begin{bmatrix} \frac{4}{3} & 1 \\ 0 & \frac{8}{3} \end{bmatrix}$

$\begin{bmatrix} h_1^{(2)} \\ h_2^{(2)} \end{bmatrix} = -\frac{9}{32}\begin{bmatrix} \frac{4}{3} & 1 \\ 0 & \frac{8}{3} \end{bmatrix}\begin{bmatrix} \frac{7}{36} \\ -1 \end{bmatrix} = \begin{pmatrix} \frac{5}{24} \\ \frac{3}{4} \end{pmatrix}$

$\therefore \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \end{bmatrix} + \begin{bmatrix} \frac{5}{24} \\ \frac{3}{4} \end{bmatrix} = \begin{bmatrix} \frac{13}{24} \\ \frac{5}{4} \end{bmatrix}$

3.3 #7 $x_{n+1} = x_n - \dfrac{f(x_n)\,(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} = \dfrac{x_n f(x_n) - x_n f(x_{n-1}) - f(x_n)x_n + f(x_n)x_{n-1}}{f(x_n) - f(x_{n-1})}$

$= \dfrac{[f(x_n)x_{n-1} - x_n f(x_{n-1})]}{f(x_n) - f(x_{n-1})}$

This is inferior to equ (3). b/c

$x_n \to x_{n+1} \approx r$, $f(x_n) \to f(x_{n+1}) \approx f(r)$

resulting in $\approx \dfrac{[f(r)\,r - r\,f(r)]}{[f(r) - f(r)]}$ "catastrophic cancellation"

Equ (3) produces $\approx r - f(r) \cdot [(r-r)/(f(r)-f(r))] \approx r$