SEC 3.2

15. $e_{n+1} = e_n - f(x_n)/f'(x_0)$. Also $0 = f(r) = f(x_n - e_n) = f(x_n) - e_n f'(\xi_n) \Rightarrow f(x_n) = e_n f'(\xi_n)$. Hence, $e_{n+1} = e_n(1 - f'(\xi_n)/f'(x_0)) \Rightarrow s = 1$, $C = 1 - f'(\xi_n)/f'(x_0)$.

17. **b.**

19. $f(r) = f'(r) = \cdots = f^{(k-1)}(r) = 0 \neq f^{(k)}(r)$.
$f(x_n) = f(r+e_n) = f(r)+e_n f'(r)+\cdots+e_n^{(k-1)} f^{(k-1)}(r)/(k-1)!+e_n^k f^{(k)}(r)/k! + e_n^{k+1} f^{(k+1)}(\xi_n)/(k+1)! \Rightarrow f(x_n) = e_n^k f^{(k)}(r)/k! + e_n^{k+1} f^{(k+1)}(\xi_n)/(k+1)!$.
Similarly, $f'(x_n) = e_n^{k-1} f^k(r)/(k-1)! + e_n^k f^{(k+1)}(\eta_n)/k!$. Then $e_{n+1} = x_{n+1} - r = x_n - r - kf(x_n)/f'(x_n) = e_n - [e_n^k f^{(k)}(r)/(k-1)! + e_n^{k+1} f^{(k+1)}(\xi_n)/(k+1)(k-1)!]/[e_n^{k-1} f^{(k)}(r)/(k-1)! + e_n^k f^{(k+1)}(\eta_n)/k!] = e_n^2[f^{(k+1)}(\eta_n)/k! - f^{(k+1)}(\xi_n)/(k+1)(k-1)!]/[f^{(k)}(r)/(k-1)! + e_n f^{(k+1)}(\eta_n)/k!]$, implying quadratic convergence.

Alternative Solution: $f^{(j)}(r) = 0$ for $0 \le j \le m-1$ and $f^{(m)}(r) \neq 0$. So the Taylor formula gives $f(r+h) = f(r) + hf'(r) + \cdots + [h^{m-1}/(m-1)!]f^{(m-1)}(r) + [h^m/m!]f^{(m)}(r) + \cdots$. Then $f(x_n) = f(r+e_n) = [e_n^m/m!]f^{(m)}(r) + e_n^{m+1}A$ where $A \equiv f^{(m+1)}(\xi_n)/(m+1)!$.

Similarly, $f'(x_n) = [e_n^{m-1}/(m-1)!]f^{(m)}(r) + e_n^m B$ where $B \equiv f^{(m+1)}(\eta_n)/m!$.

Now $e_{n+1} = x_{n+1} - r = x_n - r - mf(x_n)/f'(x_n) = e_n - mf(x_n)/f'(x_n) = [e_n f'(x_n) - mf(x_n)]/f'(x_n)$
$= \{e_n[e_n^{m-1} f^{(m)}(r)/(m-1)! + e_n^m B] - m[e_n^m f^{(m)}(r)/m! + e^{m+1} A]\}/\ \{e_n^{m-1} f^{(m)}(r)/(m-1)! + e_n^m B\}$
$= [e_n^{m+1}B - me_n^{m+1}A]/[e^{m-1} f^{(m)}(r)/(m-1)! + e_n^m B] = e_n^2\{(B - mA)/[f^{(m)}(r)/(m-1)! + e_n B]$.
We need to assume $f, f', \ldots, f^{(m+1)}$ are continuous and that $f^{(m+1)}(r) \neq 0$.

23. **a.** $J = \begin{bmatrix} 8x_1 & -2x_2 \\ 4x_2^2 - 1 & 8x_1x_1 \end{bmatrix}$. So $J(0,1) = \begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}$ and $J^{-1}(0,1) = (1/6)\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$.

Thus, $\begin{bmatrix} h_1^{(1)} \\ h_2^{(1)} \end{bmatrix} = -(1/6)\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ -1/2 \end{bmatrix}$. So $\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/2 \end{bmatrix}$.

Next $\begin{bmatrix} h_1^{(2)} \\ h_2^{(2)} \end{bmatrix} = -(1/6)\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 1/3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/6 \end{bmatrix}$.

Thus, $\begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} 1/3 \\ -1/2 \end{bmatrix} + \begin{bmatrix} 1/3 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 2/3 \\ -1/3 \end{bmatrix}$.

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3. $f(x+h) = f(x) + hf'(x) + O(h^2) \Rightarrow f'(x) \approx (f(x+h) - f(x)]/h$. Thus, $kf'(x) \approx k[f(x+h) - f(x)]/h$.
Similarly, $hf'(x) \approx h[f(x+k) - f(x)]/k$.
Thus, we obtain $kf'(x) - hf'(x) \approx [k^2 f(x+h) - h^2 f(x+k) + (h^2 - k^2)f(x)]/(kh) \Rightarrow f'(x) \approx [k^2 f(x+h) - h^2 f(x+k) + (h^2 - k^2)f(x)]/[(k-h)kh]$.

SEC 3.2 (COMPUTER PROBLEM)

13. From Computer Problem 3.2.11, we note that $f(z) = 1+z^2+e^z = 1+(x+iy)^2+e^{x+iy} = 1+x^2-y^2+2ixy+e^xe^{iy} = (1+x^2-y^2)+i(2xy)+e^x(\cos+i\sin y) = (1+x^2-y^2+e^x\cos y)+i(2xy+e^x\sin y) = f_1(x,y)+if_2(x,y)$. So it is the same problem as in Computer Problem 3.2.11.

SEC 3.3 (COMPUTER PROBLEM)

3. a Three roots 0, ± 1.391745200271.