# MA 514 HW3

3.4 #7 Eventually 0.7390851332 appears.

Let $f(x) = \cos x$, then $|\cos x - \cos y| = \sin \xi \, |x - y|$

for some $\xi$ in $(x, y)$, also $|\sin \xi| < 1$

$f(x)$ is a contraction and thus has a fixed point.

#12 Let $x_1 = \sqrt{p}$, $x_2 = \sqrt{p + \sqrt{p}} = \sqrt{p + x_1}$, $x_3 = \sqrt{x_2 + p}$

in general $x_{n+1} = \sqrt{p + x_n}$, let $f(x) = \sqrt{p + x}$, If $\lim x_n$

exists (It does, it is easy to prove this) denote it by $x$.

$\therefore \; x = \sqrt{p + x} \Rightarrow x^2 = p + x \Rightarrow x = \dfrac{1 + \sqrt{1 + 4p}}{2}$

3.5 #3 $p(4) = 946$, $p'(4) = 1808$

$$z_1 = z_0 - \frac{p(z_0)}{p'(z_0)} = 4 - \frac{946}{1808} = 3.47677$$

#10 $p(6) = 10181$, $p'(6) = 7030$

$$z_1 = z_0 - \frac{p(z_0)}{p'(z_0)} = 6 - \frac{10181}{7030} = 4.55178$$

6.1 #8 $p(x) = a + bx + cx^2$ then we have

$p(0) = a$, $p(1) = a + b + c$, $p'(\xi) = b + 2c\xi$

The determinant of the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2\xi \end{pmatrix}$ should not be zero

$\therefore \; \xi \neq \frac{1}{2}$