MA 514 HW4

6.1 #9 Let $g(x_i) = f(x_i) \quad 0 \le i \le n-1$

$h(x_i) = f(x_i) \quad 1 \le i \le n$

let $k(x) = g(x) + \frac{x_0 - x}{x_n - x_0}(g(x) - h(x))$

$\therefore k(x_0) = g(x_0) = f(x_0)$

for $1 \le i \le n-1$, $k(x_i) = g(x_i) + \frac{x_0 - x_i}{x_n - x_0}(g(x_i) - h(x_i)) = g(x_i) = f(x_i)$

$k(x_n) = g(x_n) + h(x_n) - g(x_n) = h(x_n) = f(x_n)$

#22. Lagrange: $p(x) = -\frac{1}{2}(x+2)(x-1) - \frac{1}{3}x(x+2) = -\frac{1}{6}(5x^2+7x-6)$

Newton form:

$C_0 = y_0 = 0$

$C_1 = \frac{1-0}{2} = \frac{1}{2}$

$C_2 = \frac{-1-[0+\frac{1}{2}\cdot 1\cdot 3]}{1\cdot 3} = -\frac{5}{6}$

$\therefore p(x) = \frac{1}{2}(x+2) - \frac{5}{6}(x+2)x = -\frac{1}{6}(5x^2+7x-6)$

#27 $|f(x) - p(x)| \le \frac{1}{13!}|f^{(13)}(\xi_x)| \prod_{i=0}^{12}|x - x_i|$

$f(x) = f^{(13)}(x) = e^{x-1}$, $|f(\xi)| \le f(1) = e^0 = 1$ for $\xi \in [-1,1]$

$\prod_{i=1}^{12}|x - x_i| \le 2^{13}$

$\therefore |f(x) - p(x)| \le 2^{13} \cdot 1 / 13! = 1.315 \times 10^{-6}$

6.2 #19 at x_0, we have

$$[(x_n - x_0)\,u(x_0) + (x_0 - x_0)\,v(x_0)]/(x_n - x_0) = u(x_0) = f(x_0)$$

for $1 \le i \le n-1$, we have

$$[(x_n - x_i)\,u(x_i) + (x_i - x_0)\,v(x_i)]/(x_n - x_i)$$

$$= (x_n - x_0)\,f(x_i)/(x_n - x_0) = f(x_i)$$

at x_n, we have

$$[(x_n - x_n)\,u(x_n) + (x_n - x_0)\,v(x_n)]/(x_n - x_0) = v(x_n) = f(x_n)$$

#24

x	$f(x)$			
4	63	26	6	1
2	11	2	5	
0	7	7		
3	28			

$$\therefore\ p(x) = 63 + 26(x-4) + 6(x-4)(x-2) + x(x-4)(x-2)$$