MA 514 HW5

6.3 #3 By theorem 1, there exists a unique polynomial p of degree $\le m$ ($m = 2n+1$) s.t. $p(x_i) = y_i$ and $p'(x_i) = 0$ $\quad 0 \le i \le n$

By equ (6)

$$p(x) = \sum_{i=0}^{n} y_i \left[1 - 2(x - x_i)\, l_i'(x_i)\right] l_i^2(x)$$

where $\quad l_i(x) = \prod_{j=0, j\ne i}^{n} \frac{x - x_j}{x_i - x_j} \qquad 0 \le i \le n.$

#12 Induction.

$n = 1$, $x_0 < x_1$ to prove $\frac{\partial}{\partial x_0} f[x_0, x_1] = f[x_0, x_0, x_1]$ WOLG

$$\frac{\partial}{\partial x_0} f[x_0,x_1] = \lim_{x\to x_0} \frac{f[x,x_1] - f[x_0,x_1]}{x - x_0} = \lim_{x \to x_0} \frac{(f(x_1)-f(x))/(x_1-x) - (f(x_1)-f(x_0))/(x_1-x_0)}{x - x_0}$$

$$= \lim_{x\to x_0} \left[\frac{(x_1-x_0)(f(x_0)-f(x))}{(x-x_0)(x_1-x_0)(x_1-x)} + \frac{f(x_1)-f(x_0)}{(x_1-x_0)(x_1-x)} \right]$$

$$= \frac{f(x_1)-f(x_0)}{(x_1-x_0)^2} - \frac{f'(x_0)}{x_1-x_0}$$

$$= f[x_0, x_0, x_1]$$

Now suppose it is true for $n \le k$

$n = k+1$ Case :

$$\frac{\partial}{\partial x_i} f[x_0, \cdots, x_{k+1}] = \frac{\partial}{\partial x_i}\left[\frac{f[x_1, \cdots x_{k+1}] - f[x_0, \cdots x_k]}{x_{k+1} - x_0} \right]$$

$$= \frac{f[x_1, \cdots x_i, x_i, x_{i+1}, \cdots x_{k+1}] - f[x_0, \cdots x_i, x_i, \cdots x_k]}{x_{k+1} - x_0} \quad \text{by assumption}$$

$$= f[x_0, \cdots, x_i, x_i, x_{i+1}, \cdots x_{k+1}]$$

6.3 #16 $p'(t) = -(b-a)\left[-6(b-t)/(b-a)^2 + 6(b-t)^2/(b-a)^3\right]$

$= -6(b-t)(a-t)/(b-a)^2$

So $p'\left(\frac{a+b}{2}\right) = -6\left(\frac{b-a}{2}\right)\left(\frac{a-b}{2}\right)/(b-a)^2 = \frac{3}{2}$

therefore $p'(t) \leq p'\left(\frac{a+b}{2}\right) = \frac{3}{2}$

$p(a) = b - (b-a) = a$

$p(b) = b - (b-a)\cdot 0 = b$

$p'(a) = 0$, $p'(b) = 0$

6.4 #5 $f(1^-) = 1 = f(1^+)$ $\therefore f$ is continuous at $x=1$

also $f(2^-) = 3/2 = f(2^+)$ $\therefore f$ is continuous at $x=2$

$$f'(x) = \begin{cases} 1 & x \in (-\infty, 1] \\ 2-x & x \in [1, 2] \\ 0 & x \in [2, \infty) \end{cases}$$

~~$\therefore f'(x)$~~

$\therefore f'(1^-) = 1 = f'(1^+)$ and $f'(2^-) = 0 = f'(2^+)$

$\therefore f$ is quadratic spline.