SEC 6.1

9. Let $g(x_i) = f(x_i)$ for $0 \le i \le n-1$ and $h(x_i) = f(x_i)$ for $1 \le i \le n$. Set $k(x) = g(x) + [(x_0 - x)/(x_n - x_0)][g(x) - h(x)]$. Then $k(x_0) = g(x_0) = f(x_0)$ and for $1 \le i \le n-1$ we have $k(x_i) = g(x_i) + [(x_0 - x_i)/(x_n - x_0)][g(x_i) - h(x_i)] = g(x_i) = f(x_i)$ and $k(x_n) = h(x_n) = f(x_n)$.

14. Say $x_j = 0$. $|p(x) - f(x)| = \left|(1/n!)f^{(n)}(\xi_x)\prod_{i=0}^{n-1}(x - x_i)\right|$

$\le (1.5431/n!)|x|\prod_{\substack{i \ne j \\ i=0}}^{n-1}(x - x_i) \le (1.5431/n!)|x|2^{n-1}$ since node x_j is 0. Note as in Problem 6.1.13,

$$f^{(n)}(\sinh x) = \begin{cases} \sinh x & n \text{ even} \\ \cosh x & n \text{ odd} \end{cases}$$

and $|f^{(n)}(\sinh x)| \le \max\{\sinh 1, \cosh 1\}$, on $[-1, 1]$.

So $|p(x) - f(x)|/|f(x)| \le (1.5431/n!)|x/\sinh x|2^{n-1} \le (1.5431/n!)2^{n-1} \le (2^n/n!)$

since $x/\sinh x \le 1$.

22. Lagrange form: $p(x) = -(1/2)(x+2)(x-1) - (1/3)x(x+2) = -(1/6)(5x^2 + 7x - 6)$.

Newton form:

x	$f(x)$		
-2	0	1/2	$-5/6$
0	1	-2	
1	-1		

$p(x) = (1/2)(x+2) - (5/6)(x+2)x = -(1/6)(5x^2 + 7x - 6)$.

27. $|f(x) - p(x)| \le \left|f^{(13)}(\xi_x)\right|/(13!)\prod_{i=0}^{12}|x - x_i|$. Now $f(x) = f^{(13)}(x) = e^{x-1}$ and $|f(\xi)| \le f(1) = e^0 = 1$ for $\xi \in [-1, 1]$. Also, $\prod_{i=0}^{12}|x - x_i| \le 2^{13}$. Therefore, $|f(x) - p(x)| \le 2^{13}/(13!) = 1.315 \times 10^{-6}$.

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5. The unique polynomial of degree at most n that interpolates p at $x_0, x_1, \ldots, x_n$ is p itself. Hence, the desired equation is Equation (10), with $f = p$.

6. **(Proof by induction).** For $n = 1$, $(\alpha f + \beta g)[x_0, x_1] = [(\alpha f + \beta g)(x_1) - (\alpha f + \beta g)(x_0)]/(x_1 - x_0)$ $= [\alpha f(x_1) - \alpha f(x_0)]/(x_1 - x_0) + [\beta g(x_1) - \beta g(x_0)]/(x_1 - x_0) = \alpha f[x_0, x_1] + \beta g[x_0, x_1]$. Suppose it is true for $2, 3, \ldots, n$. Consider, $(\alpha f + \beta g)[x_0, x_1, \ldots, x_{n+1}]$
$= \{(\alpha f + \beta g)[x_1, \ldots, x_{n+1}] - (\alpha f + \beta g)[x_0, \ldots, x_n]\}/(x_{n+1} - x_0)$
$= \{\alpha f[x_1, \ldots, x_{n+1}] - \alpha f[x_0, \ldots, x_n] + \beta g[x_1, \ldots, x_{n+1}] - \beta g[x_0, \ldots, x_n]\}/(x_{n+1} - x_0)$
$= \alpha f[x_0, \ldots, x_{n+1}] + \beta g[x_0, \ldots, x_{n+1}]$.

17.

x	$f(x)$			
1	3	1/2	1/3	−2
3/2	13/4	1/6	−5/3	
0	3	−4/6		
2	5/3			

Thus, $p(x) = 3 + (1/2)(x-1) + (1/3)(x-1)(x-3/2) - 2(x-1)(x-3/2)(x)$.

19. At x_0, we have $[(x_n - x_0)u(x_0) + (x_0 - x_0)v(x_0)]/(x_n - x_0) = u(x_0) = f(x_0)$. For $1 \le i \le n-1$, we have $[(x_n - x_i)u(x_i) + (x_i - x_0)v(x_i)]/(x_n - x_0) = [(x_n - x_i)f(x_i) + (x_i - x_0)f(x_i)]/(x_n - x_0)$ $= (x_n - x_0)f(x_i)/(x_n - x_0) = f(x_i)$. At x_n, we have $[(x_n - x_n)u(x_n) + (x_n - x_0)v(x_n)]/(x_n - x_0) =$ $[(x_n - x_0)/(x_n - x_0)]v(x_n) = v(x_n) = f(x_n)$.