6.5 The B-Splines: Basic Theory

7. Just let x tend to ∞ in Lemma 7, and use Lemma 4.

6.7 Taylor Series

6. $(2/\sqrt{\pi})\int_0^x e^{-t^2}dt = (2/\sqrt{\pi})\int_0^x \sum_{k=0}^{\infty}[(-t^2)^k/k!]dt$
$= (2/\sqrt{\pi})\sum_{k=0}^{\infty}[(-1)^k/k!]\int_0^x t^{2k}dt = (2/\sqrt{\pi})\sum_{k=0}^{\infty}\{[(-1)^k x^{2k+1}]/[k!(2k+1)]\}$.

14. $f(x) = -\int_0^x[(1/t)\sum_{k=1}^{\infty}(-t^k/k)]dt = \sum_{k=1}^{\infty}[\int_0^x(t^{k-1}/k)dt] = \sum_{k=1}^{\infty}(x^k/k^2)$. By Ratio Test, $\lim_{n\to\infty}|[x^{n+1}/(n+1)^2]/[x^n/n^2]| = \lim_{n\to\infty}|x[n/(n+1)]^2| = |x|$. Thus, radius of convergence is 1 if $|x|<1$. $f(-2)$ cannot be found by using the old series, since $|-2|>1$. Consider $h(t) = \ln(1-t)/t$. Taylor series about -2 is $h(t) = \sum_{k=0}^{\infty}h^{(k)}(-2)(t+2)^k/k!$.
Thus, $f(x) = -\int_0^x[\sum_{k=0}^{\infty}h^{(k)}(-2)(t+2)^k/k!]dt$ and $f(-2) = \sum_{k=0}^{\infty}[h^{(k)}(-2)2^{k+1}]/[(k+1)k!]$. $f(0.001)$ can be found using old series: $f(0.001) \approx 0.00100\,02501$.

6.8 Best Approximation: Least-Squares Theory

6. One Gram matrix has elements $a_{ij} = \int_0^1 x^i x^j\,dx = \int_0^1 x^{i+j}\,dx = (1+i+j)^{-1}$.

7. If we start with the basis $\{v_1,\ldots,v_n\}$ and apply the Gram-Schmidt process, each new vector u_j is a linear combination of $v_1,\ldots,v_j$. Hence $u_j = \sum_{i=1}^{j}a_{ij}v_i$. The coefficients a_{ij} are zero for $i>j$. Hence the matrix is upper triangular.

21. $p_0 = 1$, $p_1 = x - 1/2$, $p_2 = x^2 - x + 1/6$, $p_3 = x^3 - (23/2)x^2 + (318/30)x - (103/60)$.

22. $p_0 = 1$, $p_1 = x - a_1$, $p_2 = x^2 - (a_1+a_2)x + (a_1a_2 - b_2)$, $p_3 = x^3 - (a_1+a_2+a_3)x^2 + (a_1a_2 + a_1a_3 + a_2a_3 - b_2 - b_3)x - (a_1b_3 + a_3b_2 - a_1a_2a_3)$.