MA 514 HW 7

6.5 #3 $B_j^2(t_i) = \frac{h_{i-1}}{h_i+h_{i-1}} \delta_{i,j+1} + \frac{h_i}{h_i+h_{i-1}} \delta_{i,j+2}$

$$S(t_m) = \sum_i c_i B_i^2(t_m) = C_{m-2} B_{m-2}^2(t_m) + C_{m-1} B_{m-1}^2(t_m)$$
$$= C_{m-2} \frac{h_m}{h_m+h_{m-1}} + C_{m-1} \frac{h_{m-1}}{h_m+h_{m-1}}$$
$$= (C_{m-2}h_m + C_{m-1}h_{m-1}) / (h_m + h_{m-1}) = y_m$$

6.6 #8 $S = \sum_{j=-\infty}^{\infty} C_j B_j^3$

Thm: if $f(x) = \sum_{i=-\infty}^{\infty} c_i B_i^k(x)$, then $f'(x) = \sum_{i=-\infty}^{\infty} d_i B_i^{k-1}(x)$, where $d_i = k \frac{c_i - c_{i-1}}{t_{i+k} - t_i}$

Now $S' = \sum_{j=-\infty}^{\infty} d_j B_j^2$, where $d_j = 3 \cdot \frac{C_j - C_{j-1}}{t_{j+3} - t_j}$

$S'' = \sum_{j=-\infty}^{\infty} e_j B_j^1$, where $e_j = 2 \frac{d_j - d_{j-1}}{t_{j+2} - t_j}$

$$\therefore e_j = \frac{2}{t_{j+2}-t_j} \cdot (d_j - d_{j-1}) = \frac{6}{t_{j+2}-t_j} \left(\frac{C_j - C_{j-1}}{t_{j+3}-t_j} - \frac{C_{j-1}-C_{j-2}}{t_{j+2}-t_j} \right)$$

#9 $S''(t_i) = \sum_{j=-\infty}^{\infty} e_j B_j^1(t_i) = e_{i-1} B_{i-1}^1(t_i) = e_{i-1}$

6.8 #22 $p_0(x) = 1$

$a_1 = \langle x p_0, p_0 \rangle / \langle p_0, p_0 \rangle = \frac{1}{2}$

$\therefore p_1(x) = x - \frac{1}{2}$

$a_2 = \langle x p_1, p_1 \rangle / \langle p_1, p_1 \rangle = \frac{1}{2}$

$b_2 = \langle x p_1, p_0 \rangle / \langle p_0, p_0 \rangle = \frac{1}{12}$

$\therefore p_2(x) = x^2 - x + \frac{1}{6}$

$a_3 = \frac{1}{2}$, $b_3 = \frac{1}{15}$

$\therefore p_3(x) = x^3 - \frac{3}{2}x^2 + \frac{3}{5}x - \frac{1}{20}$

6.8 #22 Additional problem.

using $\sum_{j=0}^{n} c_j < u_j, u_i > = < f, u_i >$ $n=3$

also $< u_i, u_j > = 0 \quad \forall\, i \neq j$

$\therefore \quad c_j < u_j, u_j > = < f, u_j >$

$< f, p_0 > = \frac{2}{\pi}$, $< f, p_1 > = 0$

$< f, p_2 > = \frac{\pi^2 - 12}{3\pi^3}$

$< f, p_3 > = 0$

$\therefore \quad c_0 = \frac{< f, p_0 >}{< p_0, p_0 >} = \frac{2}{\pi}$, $c_1 = 0$

$c_2 = \frac{< f, p_2 >}{< p_2, p_2 >} = \frac{60(\pi^2 - 12)}{\pi^3}$, $c_4 = 0$

$\therefore \quad f(x) = \frac{2}{\pi} + \frac{60(\pi^2 - 12)}{\pi^3} (x^2 - x + \frac{1}{6})$