MA 514 HW9

7.3 #9 $\int_{-1}^{1} f(x)\,dx \approx c\,[f(x_0)+f(x_1)+f(x_2)]$

$f(x)=1 \Rightarrow$ LHS $=2$, RHS $= c\cdot 3 \Rightarrow c=\frac{2}{3}$

$f(x)=x \Rightarrow$ LHS $=0$, RHS $=\frac{2}{3}(x_0+x_1+x_2)$

$f(x)=x^2 \Rightarrow$ LHS $=\frac{2}{3}$; RHS $=\frac{2}{3}(x_0^2+x_1^2+x_2^2)$

let $x_1=0$, $x_0=-x_2=-\frac{1}{\sqrt{2}}$

#11 $f(x)=1 \Rightarrow$ LHS $=2$, RHS $=2$

$f(x)=x \Rightarrow$ LHS $=2$, RHS $=2$

$f(x)=x^2 \Rightarrow$ LHS $=\frac{8}{3}$, RHS $=2\alpha^2-4\alpha+4$

$f(x)=x^3 \Rightarrow$ LHS $=4$, RHS $=6\alpha^2-12\alpha+8$

$\therefore\ 3\alpha^2-6\alpha+2=0$

$\therefore\ \alpha = 1 \pm \frac{1}{\sqrt{3}}$

7.4 #6 (a) $a=1,\ b=3,\ f(x)=\frac{1}{x}$

$R(0,0)=\frac{1}{2}(b-a)[f(a)+f(b)] = \frac{4}{3}$

$R(1,0)=\frac{1}{2}R(0,0)+\frac{1}{2}(b-a)f(a+\frac{b-a}{2}) = \frac{7}{6}$

$R(2,0)=\frac{1}{2}R(1,0) + 14(b-a)[f(a+\frac{b-a}{4})+f(a+\frac{3(b-a)}{4})] = \frac{67}{60}$

$R(1,1)=\frac{4}{3}R(1,0) - \frac{1}{3}R(0,0) = \frac{10}{9}$

$R(2,1)=\frac{4}{3}R(2,0)-\frac{1}{3}R(1,0) = \frac{11}{10}$

$R(2,2)=\frac{16}{15}R(2,1)-\frac{1}{15}R(1,1) = \frac{742}{675} \approx 1.09926$

(b) $a=0,\ b=\frac{\pi}{2},\ f(x)=(\frac{x}{\pi})^2$

Similar to (a) part

$R(0,0)=\frac{\pi}{16}$, $R(1,0)=\frac{3\pi}{64}$, $R(2,0)=\frac{11\pi}{256}$

$R(1,1)=\frac{\pi}{24}$, $R(2,1)=\frac{\pi}{24}$, $R(2,2)=\frac{\pi}{24}$

7.4 #9 $I = T(f,h) + c_1 h + c_2 h^2 \cdots$ and $I = T(f, h/2) + c_1 \frac{h}{2} + c_2 (\frac{h}{2})^2 + \cdots$

$\therefore\ I = 2T(f,\frac{h}{2}) - T(f,h) + (\frac{1}{2}-1)\, c_2 h^2 + \cdots$

Let $R(1,1) = \cancel{2R}\ 2R(1,0) - R(0,0)$.

In general $R(n,1) = 2R(n,0) - R(n-1,0)$

Now $I = R(1,1) + b_2 h^2 + b_3 h^3 + \cdots$ and $I = R(2,1) + b_2 (\frac{h}{2})^2 + b_3 (\frac{h}{2})^3 + \cdots$

$\therefore\ I = \frac{4}{3} R(2,1) - \frac{1}{3} R(1,1) + (\frac{1}{2}-1)\, b_3 h^3 + \cdots$

Let $R(2,2) = \frac{4}{3} R(2,1) - \frac{1}{3} R(1,1)$

In general $R(n,2) = \frac{4}{3} R(n,1) - \frac{1}{3} R(n-1,1)$.

Now $I = R(2,2) + c_3 h^3 + c_4 h^4 + \cdots$ and $I = R(3,2) + c_3 (\frac{h}{2})^3 + c_4 (\frac{h}{2})^4 + \cdots$

$\therefore\ I = \frac{8}{7} R(3,2) - \frac{1}{7} R(2,2) + (\frac{1}{2}-1)\, c_4 h^4 + \cdots$

Let $R(3,3) = \frac{8}{7} R(3,2) - \frac{1}{7} R(2,2)$

New Eq (5) is $R(n,m) = R(n,m-1) + (\frac{1}{2^m - 1})[R(n,m-1) - R(n-1,m-1)]$