Control and Stabilization of the Korteweg-de Vries Equation

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Abstract

The Korteweg-de Vries (KdV) equation

$$u_t + uu_x + u_{xxx} = 0, \quad u \equiv u(x,t),\ -\infty < x,\, t < \infty$$

was first derived by Korteweg and de Vries in 1895 as a model for propagation of some surface water waves along a channel. It has been intensively studied from various aspects of both mathematics and physics since the 1960s when solitons were discovered through solving the KdV equation and the inverse scattering method, a so-called nonlinear Fourier transform, was invented to seek solitons. It turns out that the equation is not only a good model for some water waves but also a very useful approximation model in nonlinear studies whenever one wishes to include and balance a weak nonlinearity and weak dispersive effects.

The study of the KdV equation from control point of view began with David Russell's work, *Computational study of the Korteweg-de Vries equation with localized control action,* conducted in late 1980s. Since then much progress has been made for control theory of the KdV equation, in particular, for its controllability and stabilizability. In this talk, I will give an overview of the main results obtained in the past twenty years in this subject and demonstrate how various smoothing properties of the KdV equation have played important roles in the study. Some open problems will be presented for further investigation.