Parallel Time Integration Strategies Based on Defect Correction.

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In this talk we will discuss a new class of parallel time integrators based on Integral Deferred Correction. The need for highly accurate, highly efficient, numerical techniques for approximating the solution of initial value problems of ordinary differential equations are ubiquitous. In the setting of partial differential equations, much work has been done to utilize parallel architectures in the spatial discretization, however, there has been little mention in the literature on how to parallelize the time discretization as a means to obtain an important speedup. In recent years, "parareal" algorithms have surfaced, where the time integration interval is divided into sub-intervals and a serial prediction computation is performed, followed by a parallel corrective iteration and then a serial corrective iteration. Discussion and analysis of parareal algorithms can be found in [7, 8, 14, 15, 9].

Our approach is to construct arbitrary order time integrators based on defect correction [6, 1]. The author, with his student (Ms. Maureen Morton) and his current and former post doc's (Dr. Ben Ong and Dr. Jing-Mei Qiu) have developed Integral Defect Correction (IDC)[5, 4, 3], an extension of Spectral Deferred Correction (SDC)[6, 2, 13, 10, 11, 12]. Defect correction, like Richardson extrapolation, is a prediction–correction strategy. With each correction loop, an approximation to the error is computed and, after correction, the order of accuracy of the solution increases in a predictable manner. The methods presented in this work differ from traditional defect correction in that the integral from of the residual is used in the formulation of the solution instead of the differential form of the residual. On multi-core architectures, defect correction methods become extremely attractive because each correction step can be decoupled from the prediction and prior correction steps. In so doing, the methods are able to achieve high order accuracy in the wall clock time equivalent to that of the prediction step, provided multiple cores are used for the computation. We first present IDC, where the integral of the residual is used in the formulation of the solution. Then, we discuss a proposed modification of IDC methods, dubbed Revisionist IDC (RIDC), which allow for the corrections to be computed in parallel. The parallel framework is then presented, along with preliminary results, and a discussion of challenges to be addressed, independent of the architecture.

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