Integrate Matrix Riccati Differential Equations Whose Solutions May Have Singularities

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March 26, 2010

Abstract

Matrix Riccati Differential Equations (MRDE)

$$X' = A_{21} - XA_{11} + A_{22}X - XA_{12}X, \quad X(0) = X_0,$$

where $A_{ij} \equiv A_{ij}(t)$, appear frequently throughout applied mathematics, science, and engineering. In this talk we shall present a family of anadromic numerical methods. Besides the symmetry property which is usually preserved by most existing methods, our proposed methods also preserve two other important properties — *Bilinear Dependence* on the initial value, and *Generalized Inverse Relation* between MRDE and its complementary MRDE. By preserving the generalized inverse relation, our methods are able to accurately integrate MRDE whose solution has singularities.

Our methods are *anadromic*, meaning if a MRDE is integrated by one of our methods from $t = \tau$ to $\tau + \theta$ and then integrated backward from $t = \tau + \theta$ to τ using the same method, the value at $t = \tau$ is recovered, in the absence of rounding errors. This implies that our methods are necessarily of even order of convergence. For time-invariant MRDE, methods of any even order of convergence are established, while for time-varying MRDE, methods of order as high as 10 are established.

Our methods are semi-implicit, in the sense that there are no nonlinear systems of matrix equations to solve, only the linear ones, unlike any existing implicit methods. Given the availability of high quality code for linear matrix equations, our methods can be fairly easily implemented and embedded into any application software package that needs a robust MRDE solver.