Recent Results on Nonconforming Finite Elements

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Abstract

It has been well-known that standard simple conforming finite elements of lowest order for (Navier-)Stokes and plane elasticity problems would produce undesirable features such as instability or numerical locking. Contrary to conforming finite elements, the use of simple nonconforming finite elements of lowest order gives numerically stable solutions. Also the use of nonconforming elements draws attention in view of domain decomposition methods, such as mortar methods.

The P_1-nonconforming simplicial elements of Crouzeix-Raviart (1973) is quite a natural choice as the lowest-order elements for simplexes. However, in the quadrilateral case, it has been a kind of puzzle to choose a lowest-order nonconforming element, (including the works of Houde (1984), Rannacher and Turek (1992), Douglas et al. (1999),) especially since Arnold, Boffi, and Falk (2000) gave an equivalent condition for a quadrilateral element to have an optimal-order of convergence.

Motivated by the work of Arnold-Boffi-Falk, Park and Sheen (SIAM J NA, 2003) introduced a P_1-nonconforming quadrilateral finite element for second–order elliptic problems in two dimension that has only 3 degrees of freedom instead of 4.

Quadratic nonconforming elements on triangles are Morley element (1968) and Fortin-Soulie element (1983). The rectangular counterpart for Morley element is the incomplete biquadratic element which is of 8 degrees of freedom. Recently H. Lee and I found a counterpart of Fortin-Soulie elements on rectangles that has essentially 7 degrees of freedom, which will be announced in this lecture.

We will also report some recent success story about the application of nonconforming elements to topology optimization. In the topology optimization, the use of lowest conforming element has resulted in checkerboard phenomena. We will show the use of nonconforming elements heals these phenomena completely in both 2D (2003, Jang et al. Int. J. Num. Meth. Eng.) and 3D (to appear).

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