MATH 692: HW 1

Due date: Feb. 2

1. Write a subroutine to compute the zeroes of orthogonal polynomials defined by a three-term recurrence relations.

(i) Use the subroutine to compute the Chebyshev-Gauss points with $N = 16$ and compare it with the explicit formula in Page 17.

(ii) Use the subroutine to compute the Legendre-Gauss-Lobatto points with $N = 16$.

2. Derive the explicit formula for Chebyshev-Gauss-Radau points and weights with "-1" being a point.

3. Prove 1.3.23.

4. (optional for non-math students) Let $I = (-1, 1)$, $X_N = \{u \in P_N : u(\pm 1) = u'(\pm 1) = 0\}$ and define

$$\pi_N^{2,0} : H_0^2(I) = \{u \in H^2(I) : u(\pm 1) = u'(\pm 1) = 0\} \to X_N$$

by

$$\int_I (u - \pi_N^{2,0} u)''\, v_N'' dx = 0, \quad \forall v_N \in X_N.$$

Derive an estimate for $\|(u - \pi_N^{2,0} u)''\|_{L^2}$.

5. Problem 1 in Page 78.

6. (optional) Let $\{p_k\}$ be a sequence of orthogonal polynomials, with respect to the weight function $\omega(x)$ and the interval (a, b), generated by the three-term recurrence relation (setting $p_{-1}(x) \equiv 0$)

$$p_{k+1}(x) = (a_k x - b_k) p_k(x) - c_k p_{k-1}(x), \quad k \ge 0.$$

Find the three-term recurrence relation corresponding to $q_k(x) = \frac{p_{k+1}(x) - \alpha_k p_k(x)}{x - a}$ with $\alpha_k = \frac{p_{k+1}(a)}{p_k(a)}$ which is orthogonal with respect to the weight function $\omega(x)(x - a)$.