MATH 692: Homework 2

Due date: Feb. 19

1. Allen-Cahn Equation:

Consider the Allen-Cahn equation:

$$-u_{xx} + \frac{1}{\varepsilon^2}(u^3 - u) = f, \quad x \in (-1, 1); \quad u(\pm 1) = 0.$$

(i) Write a program for solving the above equation using a combination the collocation method and Newton's iteration. Compute the function f with the exact solution $u(x) = \sin(\pi x)$, then use this function f to test your program.

(ii) Take $f = 0$, the initial condition $u_0(x) = \sin(2\pi x)$, $\varepsilon = 0.02$ and use the Chebyshev-Gauss-Lobatto points with $n = 128$. Compute and plot the approximate solution.

2. Multi-domain collocation method:

(i) Let $a = x_0 < x_1 < \cdots < x_K = b$, and denote $I_j = (x_{j-1}, x_j)$ for $j = 1, 2, \cdots, K$. Write a program for the multi-domain collocation method using $(N_j + 1)$ points on each subinterval I_j for the problem:

$$-u_{xx} + p(x)u_x + q(x)u = f, \quad x \in (a, b); \quad u(a) = u(b) = 0.$$

Test your program with a simple exact solution.

(ii) Take $x_0 = a = -1$, $x_1 = -0.2$, $x_2 = 0$, $x_3 = 0.2$ $x_4 = b = 1$, and $p(x) = q(x) = 1$. Use your program to solve the above problem with the exact solution $u(x) = \exp(-100x^2)$. Determine a "good" combination of $\{N_j\}_{j=1}^4$ such that the maximum error at the Chebyshev-collocation points is less than 10^{-6}. Compare your result with the one-domain approach.