MATH 692: Final Project (revised! see the note below)

Due date: May 4

1. Consider Burgers Equation:

$$u_t - \nu u_{xx} + u u_x = 0, \quad x \in (-1,1); \quad u(\pm 1, t) = 0,$$

with the initial condition $u(x,0) = u_0(x)$.

(i) Write down a scheme which is second-order semi-implicit in time and Legendre-Galerkin in space; and write a program for the scheme.

(ii) Take $u(x,0) = -\sin \pi x$, and $\nu = 0.01$. Use $N = 129$ and a time step sufficiently small for the scheme to be stable. Plot the approximation solution at $t = 0, 0.2, 0.4, 0.6, 0.8, 1, 1.2$.

2. Consider Allen-Cahn Equation:

Consider the Allen-Cahn equation:

$$u_t - \Delta u + \frac{1}{\varepsilon^2}(u^3 - u) = 0, \quad (x,y) \in (-1,1)^2; \quad \frac{\partial u}{\partial n}|_{\partial\Omega} = 0,$$

with the initial condition $u(x,y,0) = u_0(x,y)$.

(i) Write down a scheme which is first-order semi-implicit in time and Chebyshev-collocation in space; and write a program for the scheme.

(ii) Take $u_0(x,y) = \tanh\left(\frac{\sqrt{x^2+y^2-0.5^2}}{\varepsilon}\right)$ with $\varepsilon = 0.04$. Use 65×65 points and Δt sufficiently small for the scheme to be stable. Plot the levelset $u(x,y,t) = 0$ for $t = 0, 1, 5, 10, 50, 100$.

Note: If you find it difficult to treat the Neumann boundary condition here, you may replace the Neumann boundary condition by the Dirichlet boundary condition: $u|_{\partial\Omega} = 1$.