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Dynamics of Regularized cavity Flow at High Reynolds Numbers

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Abstract. A numerical simulation of the unsteady incompressible flows in the unit cavity is performed by using a Chebychev-Tau approximation for the space variables. The high accuracy of the spectral methods and the condensed distribution of the Chebychev-collocation points near the boundary enable us to obtain reliable results for high Reynolds numbers with a moderate number of modes. It is found that a stationary solution always exists for Reynolds numbers up to 10000. For Reynolds numbers larger than a critical value which is between 10000 and 12000, no more steady solution is found, instead, there is a persistent oscillation indicating a Hopf bifurcation.

1. INTRODUCTION

With the rapid increase in computing power, we can now envisage to solve dynamical systems for large ranges of values of the physical parameters leading to nontrivial dynamics features: such as Hopf bifurcations, transitions to turbulence, etc. On the other hand, newly developed theoretical models, such as attractors, determining modes, inertial manifolds, etc. (see R.Temam [7] for a review of these aspects), lead to a better understanding of complex physical phenomena. Recently, Gustafson & Halasi [2] found a time periodic solution indicating a Hopf bifurcation in the rectangular driven cavity flow of aspect ratio equal to 2 at Re=10000 by integrating the unsteady Navier-Stokes equations (N.S.E). This led us to conjecture that as the Reynolds number increases to a certain critical value, the same kind of dynamical behavior as in the rectangular cavity flow might be observed in the unit cavity flow. Let us mention that very recently Bruneau & Jouron [1] observed transitions to turbulence in the unit cavity flow for a Reynolds number as low as 7500 by solving the steady N.S.E with a high resolution grid.

In this paper, we intend to solve the unsteady N.S.E by using the Chebychev-Tau discretization for the space variables. Since the Chebychev collocation points are condensed near the boundary, they are well suited to represent the boundary layers of the flow at high Reynolds numbers. In order to take advantage of the spectral accuracy, we consider a regularized driven cavity flow (see description below). It turns out that the regularized driven cavity flow preserves qualitatively the dynamical features of the driven cavity flow.

2. NUMERICAL SCHEME

We are going to solve the 2-D unsteady Navier-Stokes equations in the primitive variable formulation:

$$\frac{\partial u}{\partial t} - \frac{1}{Re}\Delta u + (u \cdot \nabla)u + \nabla p = 0 \tag{1}$$

$$divu = 0 \tag{2}$$

in the cavity $\Omega = (0,1) \times (0,1)$ with $u = ((1-(2x-1)^2)^2, 0)$ on the upper lid (instead of $u = (1,0)$ for the driven cavity flow) and $u = (0,0)$ on other parts of the boundary.

The equations (1-2) are discretized by using the following second order projection (or fractional) scheme, proposed by Kim & Moin [4]:

$$\frac{1}{\Delta t}(u^* - u^n) - \frac{1}{2Re}\Delta(u^* + u^n) = -\frac{1}{2}(3(u^n \cdot \nabla)u^n - (u^{n-1} \cdot \nabla)u^{n-1}) \tag{3}$$

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$$u^*|_{\partial\Omega} = u((n+1)\Delta t) + \Delta t \nabla \phi^n \tag{4}$$

and

$$\frac{1}{\Delta t}(u^{n+1} - u^*) + \nabla \phi^{n+1} = 0 \tag{5}$$

$$div u^{n+1} = 0 \tag{6}$$

$$u^{n+1}|_{\partial\Omega} = u((n+1)\Delta t) \tag{7}$$

In the first step, we solve an intermediate velocity u^* which is not physical. Then, in the second step we project u^* onto the divergence free space to get an adequate velocity approximation u^{n+1}. We note also that ϕ^{n+1} in the scheme is not a proper approximation of the original pressure. The proper approximation for the pressure is

$$p^{n+1} = \phi^{n+1} - \frac{\Delta t}{2Re} \Delta \phi^{n+1}$$

By applying the divergence operator to (5), we find that (5-7) is equivalent to (assuming $\phi^0 = 0$)

$$\Delta \phi^{n+1} = \frac{1}{\Delta t} div u^* \tag{8}$$

$$\frac{\partial \phi^{n+1}}{\partial n}|_{\partial\Omega} = 0 \tag{9}$$

$$u^{n+1} = u^* - \Delta t \nabla \phi^{n+1} \tag{10}$$

Note that (3-4) and (8-9) are both Helmholtz equations which can be approximated by the Chebychev-Tau formulation (see for instance [5]). The resulting system can be solved by the efficient diagonalization procedure (see D.B.Haidvogel & T.A.Zang [3]).

The scheme is only conditionally stable due to the explicit treatment of the nonlinear term. Our experiments showed that the time step was not very restrictive with respect to the Reynolds number and the number of space discretization modes (see [6]).

3. Numerical results

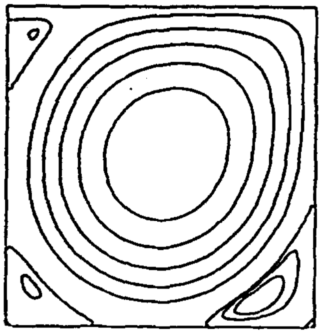
By integrating the unsteady Navier-Stokes equations (1-2) with the regularized boundary conditions, we found stationary solutions for Reynolds numbers up to 10000 . The streamlines at Re=5000 (with 32×32 modes) and at Re=10000 (with 48×48 modes) are presented in figure 1. We notice that the vortex dynamics of the regularized driven cavity flow is similar to that of the driven cavity flow.

For $Re \geq 12000$, no more steady solution was found, instead, the same kind of dynamical behavior as presented in [2] appeared. In fact, after a long smoothing time, the flow finally became quasi-periodic in time (see figure 2) which indicated a Hopf bifurcation. The final oscillation of the streamlines at Re=15000 (with 64×64 modes) is presented in figure 3. One can see clearly the appearance and the disappearance of the small vortices along the left side wall and the bottom of the cavity.

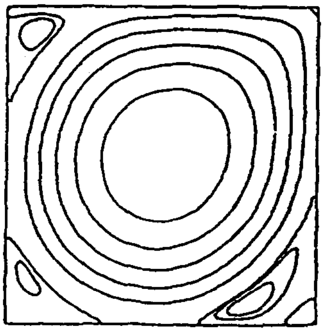
In order to double-check our results, we also implemented a standard semi-implicit time discretization scheme which required solving a generalized Stokes equation at each time step; to do this, we used the influenced matrix technique. All the above results were reproduced. A detailed presentation will be given in [6].

Figure 1: streamlines. (1-a) Re=5000; (1-b) Re=10000.

Figure 2: Oscillation of the fist component of $u(0.048, 0.01)$ at Re=15000 from 1460 second to 1500 second.



(1-a)



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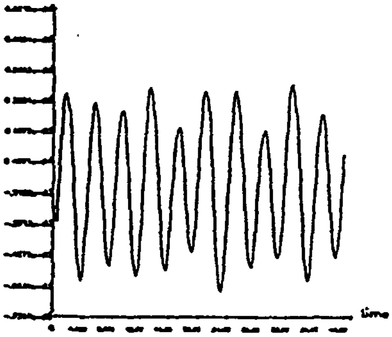
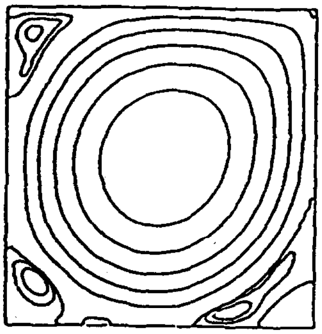


(2)

Figure 3: Final oscillation of the streamlines at Re=15000.



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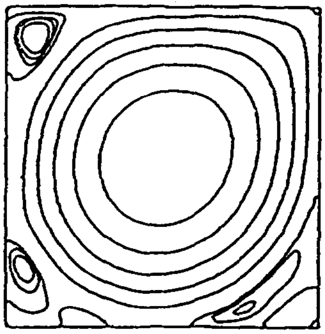

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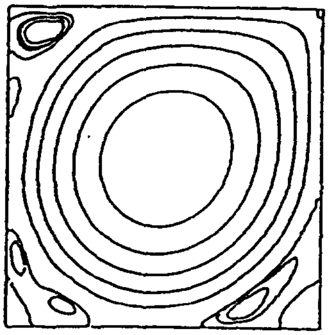

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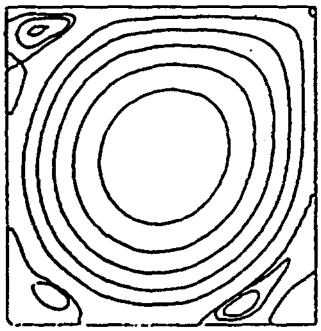

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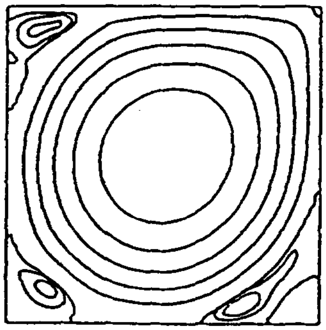

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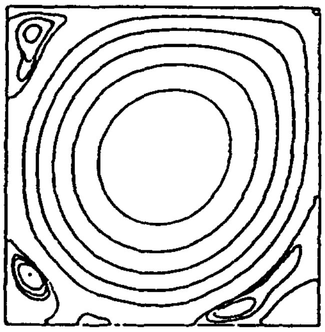

t=1503.6

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