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Error estimates for time discretizations of Cahn–Hilliard and Allen–Cahn phase-field models for two-phase incompressible flows

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Abstract We carry out rigorous error analysis for some energy stable time discretization schemes developed in Shen and Yang (SIAM J Sci Comput 32(3):1159–1179, 2010) for a Cahn–Hilliard phase-field model and in Shen and Yang (Chin Ann Math Ser B 31(5):743–758, 2010) for an Allen–Cahn phase-field model.

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1 Introduction

Phase-field approach for multi-phase incompressible flows has attracted much attention in recent years (cf. [1,13,14,16–18,24] and the references therein). For two-phase incompressible flows, the phase-field models consist of either a Navier–Stokes–Cahn–Hilliard (NSCH) system or a Navier–Stokes–Allen–Cahn (NSAC) system. These are coupled nonlinear systems which are difficult to deal with numerically. Thus, designing efficient and accurate numerical methods for solving these coupled equations has been a great challenge to the scientific computing community.

While various convergence results and error estimates are available for the Navier–Stokes equations [9,11,22], there are only a few convergence results available for phase-field models of multi-phase flows. In [5], Feng proved the convergence of dis-

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