



# A new spectral method for numerical solution of the unbounded rough surface scattering problem



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## ABSTRACT

A new spectral method is developed to solve the unbounded rough surface scattering problem. An unbounded rough surface is referred to as a non-local perturbation of an infinite plane surface such that the whole rough surface lies within a finite distance of the original plane. The method uses a transformed field expansion to reduce the boundary value problem with a complex scattering surface into a successive sequence of transmission problems of a planar surface. Hermite orthonormal basis functions are adopted to further simplify these problems to fully decoupled one-dimensional two-point boundary value problems, which are solved efficiently by the Legendre–Galerkin method. Numerical results indicate that the method is efficient, accurate, and well-suited for solving the scattering problem by unbounded rough surfaces.

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## 1. Introduction

The problems of acoustic and electromagnetic scattering by unbounded rough surfaces have received much attention from both the engineering and mathematical communities for their important applications in a wide range of scientific areas, such as modeling acoustic and electromagnetic wave propagation over outdoor ground and sea surfaces [29], optical scattering from the surface of materials in near-field optics or nano-optics [13], detection of underwater mines, especially those buried in soft sediments [34]. An unbounded rough surface is referred to as a non-local perturbation of an infinite plane surface such that the whole rough surface lies within a finite distance of the original plane. Due to the non-local perturbation, precise modeling and accurate computing present challenging mathematical and computational questions.

Mathematically, the well-posedness of the solution was studied in [9,10,12,22,25] for the acoustic wave scattering problem. In [24], a model problem was considered for the three-dimensional electromagnetic wave scattering by rough surfaces. The two-dimensional scalar model problem was also considered by integral equation methods in [8,11,15–17,36,37]. We refer to [7,30] for related scattering problems where weighted Sobolev spaces were considered for unbounded domains. In addition, the solutions are available by using approximate, asymptotic, or statistical methods in [14,19,29,31,34,35] and the references cited therein. Despite the large amount of work done so far, we are not aware of any efficient and accurate numerical method for solving the scattering problem by unbounded rough surfaces.

The present work is concerned with the numerical solution for such a scattering problem. We propose, as the first time, a new spectral method to rigorously solve the unbounded rough surface scattering problem. Specifically, we consider the

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