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CORRIGENDUM: FOURIER SPECTRAL APPROXIMATION TO A DISSIPATIVE SYSTEM MODELING THE FLOW OF LIQUID CRYSTALS*

QIANG DU†, BENYU GUO‡, AND JIE SHEN§

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The purpose of this note is to correct an error in the proof of Proposition 2.4 in [1]. The inequality $||g(d_M)||_1 \leq c||d_M||^2_{L^4}|d_M|_1$ on line 18 of page 741 in [1] is not correct. We now revise the proof and the result of Proposition 2.4 as follows. Indeed,

$$||g(d_M)||_1^2 \leq c \int_\Omega |d_M|^4 (\nabla d_M)^2 dx.$$

By integration by parts, the Cauchy inequality, and (2.10) in [1], we obtain

$$||g(d_M)||_1^2 \leq c||d_M||^5_{L^{10}}|d_M|_2 \leq c||d_M||^5_{\frac{2n}{5}}|d_M|_2 \leq cM^{2n-5}||d_M||_1^5|d_M|_2.$$

Thus, by (2.18) of [1], we have

$$||(P_M - I)g(d_M)|| \leq cM^{\frac{2n-7}{2}}||d_M||_1^{\frac{5}{2}}|d_M|_2^{\frac{1}{2}}.$$

Next, by virtue of the imbedding inequality and (2.10) of [1],

$$\begin{aligned}
2\lambda|G| &\leq 2\lambda||u_M||_{L^3}||\nabla d_M||_{L^6}||(P_M - I)g(d_M)|| \leq c\lambda M^{\frac{2n-7}{2}}||u_M||_{\frac{n}{6}}||d_M||_1^{\frac{5}{2}}||d_M||_2^{\frac{3}{2}} \\
&\leq c\lambda M^{\frac{2n-7}{2}}||u_M||^{\frac{3}{4}}_{\frac{n}{6}}||u_M||_1^{\frac{1}{4}}||d_M||_1^{\frac{5}{2}}||d_M||_2^{\frac{3}{2}} \\
&\leq c\lambda M^{\frac{9n-28}{8}}||u_M||^{\frac{3}{4}}||u_M||_1^{\frac{1}{4}}||d_M||_1^{\frac{5}{2}}||d_M||_2^{\frac{3}{2}} \\
&\leq c\lambda M^{\frac{9n-28}{40}}||u_M||_1^{\frac{1}{4}} \cdot M^{\frac{3(9n-28)}{80}}||d_M||_2^{\frac{3}{2}} \cdot M^{\frac{3(9n-28)}{160}}||u_M||^{\frac{3}{4}} \cdot M^{\frac{9n-28}{16}}||d_M||_1^{\frac{5}{2}} \\
&\leq c\lambda(M^{\frac{9n-28}{5}}||u_M||_1^2 + M^{\frac{9n-28}{20}}||d_M||_2^2 + M^{\frac{3(9n-28)}{10}}||u_M||^{12} \cdot M^{9n-28}||d_M||_1^{40}).
\end{aligned}$$

On the other hand, we have

$$\begin{aligned}
2\lambda \int_\Omega F(d_M)dx &\geq \frac{\lambda}{2\varepsilon^2}(||d_M||^4_{L^4} - 2||d_M||^2 + (2\pi)^n) \\
&\geq \frac{\lambda}{2\varepsilon^2}\left(\frac{1}{(2\pi)^n}||d_M||^4 - 2||d_M||^2 + (2\pi)^n\right) \\
&\geq \frac{\lambda}{2\varepsilon^2(2\pi)^n}(||d_M||^2 - (2\pi)^n(1+\varepsilon^2))^2 + \lambda||d_M||^2 - \frac{\lambda(2\pi)^n}{2}(2+\varepsilon^2) \\
&\geq \lambda||d_M||^2 - \tfrac{\lambda}{2}(2\pi)^n(2+\varepsilon^2).
\end{aligned}$$

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†Department of Mathematics, Penn State University, 307 McAllister Bldg., State College, PA 16802 (qdu@math.psu.edu).
‡Department of Mathematics, Shanghai Normal University, 100 Guilin Road, Shanghai 200234, People's Republic of China, (byguo@guomai.sh.cn).
§Department of Mathematics, Purdue University, West Lafayette, IN 47907 (shen@math.purdue.edu).

Moreover, by (2.23) of [1],

$$\lambda\gamma||\Delta d_M - P_M f(d_M)||^2 = \lambda\gamma\left(|d_M|_2^2 + ||P_M f(d_M)||^2 - \frac{2}{\varepsilon^2}|d_M|_1^2\right) \geq \lambda\gamma|d_M|_2^2 - \frac{2\lambda\gamma}{\varepsilon^2}|d_M|_1^2.$$

Substituting the above three estimates into (2.17) of [1] and integrating the resulting inequality with respect to t, we find that for $n \leq 3$ and M sufficiently large

(1)

$$\begin{aligned}\widetilde{E}(t) &\equiv E(t) + \int_0^t \left(\frac{\nu}{4}|u_M(s)|_1^2 + \lambda\gamma||\Delta d_M(s) - P_M f(d_M(s))||^2\right)ds \\ &\leq \sigma_0 + \int_0^t \left(\frac{2\lambda\gamma}{\varepsilon^2}||d_M(s)||_1^2 + c_4 M^{\frac{3}{10}(9n-28)}||u_M(s)||^{12} + c_4 M^{9n-28}||d_M(s)||_1^{40}\right)ds,\end{aligned}$$

where

$$\begin{aligned}&E(t) = ||u_M(t)||^2 + \lambda||d_M(t)||_1^2, \\ (2)\qquad &\sigma_0 = ||u_0||^2 + \lambda|d_0|_1^2 + 3\lambda\int_\Omega F(d_0)dx + \lambda(2\pi)^n(\varepsilon^2 + 1 + \varepsilon\sqrt{\varepsilon^2+1}).\end{aligned}$$

Finally, we apply Lemma 2.3 of [1] to the above inequality to obtain for $n \leq 3$

$$(3)\qquad \widetilde{E}(t) \leq \sigma_0 e^{\left(\frac{2\lambda\gamma}{\epsilon^2} + c_4 M^{-\frac{3}{10}}\right)t}.$$

In fact, we can derive improved results for Proposition 2.4 in the two-dimensional case (i.e., $n = 2$). Indeed, using the imbedding theory and (2.9) and (2.10) in [1], we obtain for any $\delta > 0$

$$||g(d_M)||_1 \leq c||d_M||_{L^\infty}^2|d_M|_1 \leq c||d_M||_{1+\frac{\delta}{2}}^2|d_M|_1 \leq cM^{\frac{\delta}{2}}||d_M||_1^3.$$

Thus, by (2.18) of [1],

$$||(P_M - I)g(d_M)|| \leq cM^{\frac{\delta-2}{2}}||d_M||_1^3.$$

By virtue of imbedding theory and the Cauchy inequality,

$$\begin{aligned}2\lambda|G| &\leq ||u_M||_{L^\infty}||d_M||_1||(P_M - I)g(d_M)|| \leq cM^{\delta-1}||u_M||_1||d_M||_1^4 \\ &\leq \frac{\nu}{2}|u_M|_1^2 + c_4 M^{\frac{1}{2}(\delta-1)}||u_M||^2 + c_4 M^{\frac{3}{2}(\delta-1)}||d_M||_1^8.\end{aligned}$$

Using the above estimate instead of (2.19) in [1] and repeating the same procedure as in the proof of Proposition 2.4, we obtain the following revised result.

PROPOSITION 2.4 (revised). *Let $\widetilde{E}(t)$, $E(t)$, and σ_0 be defined in* (1)–(2). *Then, for $n = 3$, we have*

$$\widetilde{E}(t) \leq \sigma_0 e^{\left(\frac{2\lambda\gamma}{\epsilon^2} + c_4 M^{-\frac{3}{10}}\right)t};$$

for $n = 2$, we have for any small $\delta > 0$,

$$\begin{aligned}&E(t) + \int_0^t \left(\frac{\nu}{2}|u_M(s)|_1^2 + 2\lambda\gamma||\Delta d_M(s) - P_M f(d_M(s))||^2\right)ds \leq \sigma_0 e^{c_4 M^{\frac{1}{2}(\delta-1)}t}, \\ &E(t) + \int_0^t \left(\frac{\nu}{2}|u(s)|_1^2 + 2\lambda\gamma|d_M(s)|_2^2\right)ds \leq \left(1 + \frac{4\gamma}{\varepsilon^2}\right)\sigma_0 e^{c_4 M^{\frac{1}{2}(\delta-1)}t}.\end{aligned}$$

Remark 1. The revised result improves the result of Proposition 2.4 in [1] when $n = 2$. We can use the revised Proposition 2.4 to prove directly the existence of a global solution for (2.3) when $n = 2$, and of a local solution for (2.3) when $n = 3$. We can also use the same techniques as in [2] to prove the existence of a global solution for (2.3) when $n = 3$.

Remark 2. There is a similar error in the proof of Theorem 3.1: the estimate (3.20) is not correct. However, we can revise the proof for Theorem 3.1 as above and show that the result of Theorem 3.1 still holds.

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