Research Highlights

The following are some highlights of Jie Shen's research accomplishments. The quoted citation numbers are from Google Scholars as of Oct. 25, 2022.

1 Analysis and development of splitting methods for incompressible Navier-Stokes equations

The projection type methods for incompressible Navier-Stokes equations, originally proposed by Chorin (1968, cited over 7100 times) and Temam (1969, cited over 1300 times) have played and are still playing a dominating role in numerical approximations of incompressuble flows. Shen has made several pioneer contributions (described below) to the analysis and development of projection type methods for the Navier-Stokes equations.

Projection and pressure-correction schemes

Rigorous and optimal error estimates for the classical projection method had been elusive for over twenty years since its inception and the lack of which had seriously hindered the developments of new and improved projection type schemes. In his 1992 paper in SIAM J. Numer. Anal. (cited over 320 times), Shen derived the first rigorous error estimate for the classical projection method and several of its popular variants. The analysis also clarified a long standing question over whether the incompatible Neumann boundary condition for the pressure in projection methods would lead to convergent pressure approximations. This paper established a general framework for the analysis of projection-type schemes and have revitalized interests in developing and analyzing new projection type schemes.

Velocity-correction schemes

Another class of projection type methods is proposed by Karniadakis, Israeli & Orszag (1991, cited over 1700 times). The boundary condition of pressure in these schemes involves second derivatives of the velocity, which makes the schemes extremely difficult to analyze and almost impossible to implement by finite-element discretization. In fact, no rigorous stability and convergence analysis were available for nearly two decades. In a 2003 paper in SIAM J. Numer. Anal. (cited over 260 times), Guermond and Shen proposed and analyzed a new class of projection methods with velocity-correction. It was shown that, with a proper reformulation, the scheme proposed by Karniadakis, Israeli & Orszag (1991) is equivalent to the velocity-correction method. Hence, they essentially solved the long standing problem associated with the scheme of Karniadakis, Israeli & Orszag (1991).

A class of truly consistent splitting schemes

None of the previous projection type schemes are truly consistent in the sense that the pressure approximations always suffer from the splitting error which had been believed to be unavoidable. In a 2003 paper in J. Comput. Phys. (cited over 210 times), Guermond and Shen developed a new class of truly consistent splitting schemes for incompressible flows. It is shown that the consistent splitting schemes are free of splitting errors and provide full accuracy. In particular, the second-order consistent splitting scheme was the only decoupled, unconditionally stable, truly second-order (for the velocity and the pressure in both L^2 and H^1 norms) scheme, and is now widely used by researchers in computational fluid dynamics.

2 Algorithm development and analysis of efficient spectral methods

A main focus of Shen's work is on the construction, analysis and application of efficient spectral methods for partial differential equations.

Fast spectral-Galerkin methods for elliptic equations

For many years, the spectral-Galerkin method had been virtually ignored by practitioners because the lack of efficiency and flexibility compared to the spectral-collocation method. In a sequence of four papers published in SIAM J. Sci. Comput. (from 1994 to 1999 with the first two cited over 670 and 290 times, respectively), Shen developed a class of fast spectral-Galerkin algorithms with quasi-optimal computational complexity for elliptic equations. They enjoy the following desired properties: (i) optimal convergence rate as allowed by the regularity of the underlying problem; (ii) well-conditioned and sparse, if the coefficients are constants or polynomials, linear system which can be efficiently solved. Hence, they are more efficient and accurate than the popular spectral-collocation method, and it is widely acknowledged that this sequence of papers have effectively revitalized the spectral-Galerkin method.

Efficient and accurate spectral method for fractional differential equations (FDEs)

The two main difficulties for dealing with FDEs are (i) fractional derivatives are non-local operators; (ii) fractional derivatives involve singular kernel/weight functions, and the solutions of FDEs are usually singular near the boundaries. Most of the existing numerical methods for FDEs are based on finite difference/finite element methods which lack the capability to effectively deal with the aforementioned difficulties, as they are based on local operations, and are not well-suited for problems with singular kernels/weights. In a 2016 joint paper in Math. Comp. (cited over 230 times), Shen and collaborators introduced a new class of generalized Jacobi functions (GJFs), which include the fracto-polynomials introduced by Karniadakis and Zayernoori in 2014 as a special case, and constructed efficient and accurate Petrov-Galerkin methods for a class of FDEs. The GJFs offer a perfect solution for the two main difficulties mentioned above, at least for a class of FDEs. This paper established a solid mathematical foundation for using GJFs to solve FDEs, and inspired a large number of subsequent work.

3 Modeling and algorithm development of phase-field models for multiphase complex fluids

An emerging scientific challenge is how to model and simulate the dynamics of complex fluids such as liquid crystals, polymers, mixture of two immersible fluids, etc. Shen made pioneering contributions in developing flexible and robust phase field models/schemes to simulate multiphase complex flows. In a 2003 paper in Physica D (cited over 620 times), Shen, joint with Liu, proposed a phase-field model for the mixture of two incompressible Newtonian fluids, and developed a very efficient and robust numerical schemes for solving the coupled nonlinear equations and presented convincing numerical examples which exhibited various physical mechanisms of the model and demonstrated its robustness and versatility. In a 2004 joint paper in J. Fluid mech. (cited over 950 times), Shen and collaborators developed a variational phase-field model and an efficient numerical scheme for multiphase complex fluids, and used it to study the complex rheology of mixtures of non-Newtonian fluids. These two papers have popularized the phase-field methods and are now standard references in the field.

4 Algorithm development and analysis of efficient numerical schemes for gradient flows

Many physical processes can be modeled with gradient flows which are usually nonlinear dissipative systems with non-convex free energies. A main difficulty is how to construct *simple, efficient and accurate* numerical schemes which are also energy dissipative. Countless efforts have been devoted to overcoming this difficulty since 1980, but none possesses all desirable properties. In a 2018 joint paper in J. Comput. Phys. (cited over 471 times), Shen and collaborators developed a surprisingly effective strategy, the so-called scalar auxiliary variable (SAV) approach, for general gradient flows which are frequently found in computational materials science and computational fluid dynamics. This ingenious approach overcomes the main drawbacks in existing schemes and enjoys the following advantages:

- it leads to a sequence of decoupled equations with constant-coefficients to solve at each time step;
- it is unconditionally energy stable and extendable to high orders; and
- it applies to a much more general class of gradient flows, including those with unbounded energy density but bounded energy.

Furthermore, in a 2018 joint paper in SIAM J. Numer. Anal., he developed a complete convergence and error analysis for the SAV approach without the Lipschitz condition (which was not satisfied with even the double-well potential) which was assumed for existing schemes with explicit treatment of part or all nonlinear terms. In a 2019 paper in *SIAM Review* (cited over 310 times), Shen and collaborators applied the SAV approach to a variety of challenging problems which can not be easily dealt with by existing approaches. This approach is truly revolutionary and attracted a large number of followup work, as it provides a *simple, efficient and accurate* method for a large class of challenging problems.